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Physics course handout

Intended for first-year students (Common Core) in Natural and Life Sciences (L1)

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General Introduction

This physics course handout is intended for first-year students (Common Core) in Natural and Life Sciences. It aligns with the programmed content for this module. The content of the Physics module is studied by first-year (Common Core) Natural and Life Sciences students during the second semester and falls under the methodological unit with the code (UEM 2.1), a coefficient of 3 and 5 credits.

The aim is to enable students to acquire fundamental knowledge in the field of physics that is closely related to the field of Natural and Life Sciences.

Having taught the Physics module for several years, I've focused on brevity in preparing this handout to align with the interests of first-year (Common Core) Natural and Life Sciences students. This was done without sacrificing the curriculum's content, and I've included examples to simplify the information. Furthermore, each chapter concludes with a set of solved exercises to reinforce concepts and understanding.

This handout contains five chapters, which we summarize as follows:

- 1- Mathematical reminder.
- 2- Optics.
- 3- Fluid mechanics.
- 4- Concept of crystallography.
- 5- Concept of spectral analysis.

Ultimately, I hope this handout helps students grasp the module and teachers provide their course. Thanks to anyone who gives suggestions to improve it.

Chapter 1

Mathematical reminder

1.1 Introduction

In this chapter, we will begin by introducing some fundamental concepts of measurement. Following this, we will explore the dimensional analysis of physical quantities, highlighting its importance in verifying the correctness of physical laws and the mathematical equations that govern physical phenomena. Furthermore, we will discuss the inherent relationship between the measurement of physical quantities and errors in measurement, examining their causes. Finally, we will conclude by detailing the calculation of errors arising from both direct and indirect measurements and how to handle them to achieve measurement accuracy.

1.2 Physical quantities and dimensional analysis

1.2.1 Physical quantities

A physical quantity is a parameter to which a measurement can be associated.

Examples: width, mass, time, force, etc.

Measurement of a physical quantity

Any quantification (measurement) of a physical quantity is done by comparison with a reference quantity (Figure :1.1).



Figure 1.1: Measurement instruments.

Unit

A unit represents a reference that allows the measurement to be made.

1.2.2 Dimension

By definition, a physical quantity G has a dimension if its measurement depends on the choice of the measurement standard. Its dimension is noted $[G]$.

Note

The dimension reflects the nature of this quantity.

Base unit, dimension and international system (SI)

The definitions of the base units (legal units) are based on the International System of Units (SI). The International System of units (SI) consists of seven base units corresponding to a physical quantity and a dimension:

Physical Quantity	Dimension	Unit	Symbol
Length	L	meter	m
Mass	M	kilogram	kg
Time	T	second	s
Temperature	θ	kelvin	K
Electric current	I	ampere	A
Luminous intensity	J	candela	cd
Amount of substance	N	mole	mol

Examples: Dimensional analysis.

* **Velocity** (v):

$$v = \frac{dx}{dt}, \quad [v] = \frac{[x]}{[t]} = LT^{-1}, \quad (1.1)$$

* **Acceleration (a):**

$$a = \frac{dv}{dt}, \quad [a] = \frac{[v]}{[t]} = LT^{-2}, \quad (1.2)$$

* **Force (F) :**

$$F = ma, \quad [F] = [m][a] = MLT^{-2}, \quad (1.3)$$

* **Ideal gas constant (R):**

Ideal gas law writes;

$$pv = nRT, \quad [R] = \frac{[p][v]}{[n][T]} = ML^2T^{-2}\theta^{-1}N^{-1}. \quad (1.4)$$

Verifying a Formula

A physical law imposes a constraint: it must be homogeneous, i.e., consist of terms of the same dimension.

Example: (Velocity (v)).

$$v = \sqrt{2gh}, \quad [v] = [g]^{\frac{1}{2}}[h]^{\frac{1}{2}} = (LT^{-2})^{\frac{1}{2}}(L)^{\frac{1}{2}} = LT^{-1}. \quad (1.5)$$

1.3 Error Calculation

Every measurement is affected by an error.

1.3.1 Error

The measurement error is defined as the difference between the measured value and the true value, which remains unknown.

1.3.2 Origin of different types of errors

Systematic errors

Due to a lack of accuracy of the instruments.

Random errors

Due to a lack of sensitivity and fidelity of a device, as well as the imperfection of the observer's senses.

1.3.3 Absolute uncertainty

Absolute uncertainty Δx is the maximum error value;

$$\Delta x = \max \left(\left| x_i - \bar{x} \right| \right), \quad (1.6)$$

where $\bar{x}$ is the average value:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}, \quad (1.7)$$

with n is the number of measurements.

1.3.4 Measurement result

Measurement result is written in the form;

$$x = \left(\bar{x} - \Delta x \right) \text{ unite}, \quad (1.8)$$

1.3.5 Relative uncertainty

Relative uncertainty is a pure number (without unit) always much smaller than 1, which is expressed in %. The relative uncertainty calculated from

$$\frac{\Delta x}{\bar{x}} 100\%. \quad (1.9)$$

1.3.6 Calculation of relative uncertainty

Relative uncertainty is calculated according to the operation between physical quantities:

Operation	x	$\frac{\Delta x}{x}$
Sum	$a + b$	$\frac{\Delta a + \Delta b}{a + b}$
Difference	$a - b$	$\frac{\Delta a + \Delta b}{a - b}$
Product	$a \times b$	$\frac{\Delta a}{a} + \frac{\Delta b}{b}$
Quotient	$\frac{a}{b}$	$\frac{\Delta a}{a} + \frac{\Delta b}{b}$
Power	a^n	$ n \frac{\Delta a}{a}$

1.4 Exercises

Exercise 1:

The following are the measured values of the velocity v ;

$v(ms)$	1	0.9	1.1	0.8	1.2	1	0.7
---------	---	-----	-----	-----	-----	---	-----

1/ Express the measurement result of the velocity v .

2/ Deduce the relative uncertainty.

Exercise 2:

We measure the surface area S of an A4 sheet, and the following measurement results for the width (b) and length (a) are written down;

$$a = (29.7 \pm 0.1) \text{ cm}, \quad (1.10)$$

$$b = (21.1 \pm 0.1) \text{ cm}, \quad (1.11)$$

1/ Express the measurement result of the surface S .

2/ Deduce the relative uncertainty of the surface S .

Exercise 3:

Determine the dimension of force (F) and pressure (P) from Newton's law;

$$F = ma, \quad (1.12)$$

and the law of pressure;

$$P = \frac{F}{S}, \quad (1.13)$$

where m : Mass, a : Acceleration and S : Surface.

Exercise 4:

The velocity of a spherical object moving in water given by;

$$v = \frac{2}{9} \rho_e (d - 1) \frac{r^2 g}{\eta}. \quad (1.14)$$

Find the dimension of the viscosity η ?

where

ρ_e : The mass density of water.

d : Density.

r : The radius of the spherical object.

g : The acceleration due to gravity.

Exercise 5:

Find the dimension of the diffusion coefficient D using Fick's law;

$$\frac{dn}{dt} = -DS \frac{\Delta c}{\Delta x}, \quad (1.15)$$

where

S : Membrane surface area.

Δx : Thickness.

$\frac{dn}{dt}$: The molar flow rate.

Δc : The concentration difference.

1.5 Solutions

Exercise 1:

1/ The measurement result of the velocity v ;

* The average value of the velocity $\left(\bar{v}\right)$:

$$\bar{v} = \frac{\sum_{i=1}^n v_i}{n}, \quad (1.16)$$

$$= \frac{1 + 0.9 + 1.1 + 0.8 + 1 + 0.7}{6}, \quad (1.17)$$

$$\simeq 0.92 m/s. \quad (1.18)$$

2/ Absolute uncertainty (Δv);

Absolute uncertainty (Δv) is the maximum error value (1.6);

$$\Delta v = \max \left(|v_i - \bar{v}| \right), \quad (1.19)$$

$$= |0.7 - 0.92|, \quad (1.20)$$

$$= 0.22 m/s. \quad (1.21)$$

* Measurement result is written in the form (1.8);

$$v = \left(\bar{v} - \Delta v \right) m/s, \quad (1.22)$$

$$= (0.92 \pm 0.22) m/s. \quad (1.23)$$

2/ The relative uncertainty:

The relative uncertainty calculated from (1.9);

$$\frac{\Delta v}{v} 100\% = \frac{0.22}{0.92} 100\%, \quad (1.24)$$

$$= 23.91\%. \quad (1.25)$$

Exercise 2:

1/ The measurement result of the surface S :

* Average result ($\bar{S}$);

$$\bar{S} = \bar{a} \times \bar{b}, \quad (1.26)$$

$$\bar{S} = (29.7 cm) \times (21.1 cm), \quad (1.27)$$

$$= 626.67 cm^2. \quad (1.28)$$

* Absolute uncertainty (ΔS);

$$\Delta S = S \left[\frac{\Delta b}{b} + \frac{\Delta a}{a} \right], \quad (1.29)$$

$$= 626.67 \text{ (cm}^2\text{)} \left[\frac{0.1}{21.1} + \frac{0.1}{29.7} \right], \quad (1.30)$$

$$= 5.08 \text{ cm}^2. \quad (1.31)$$

* The measurement result of the surface (S);

$$S = (\bar{S} \pm \Delta S) \text{ unite}, \quad (1.32)$$

$$S = (626.67 \pm 5.08) \text{ cm}^2. \quad (1.33)$$

2/ Relative uncertainty ($\frac{\Delta S}{\bar{S}}$):

$$\frac{\Delta S}{\bar{S}} = \frac{5.08}{626.67} \times 100 = 0.81\%. \quad (1.34)$$

Exercise 3:

1/ The dimension of the force (F);

$$[F] = [m] [a], \quad (1.35)$$

$$= M.L.T^{-2}. \quad (1.36)$$

where

$$a = \frac{d^2 x}{dt^2}, \quad (1.37)$$

$$[a] = \frac{[x]}{[t]^2} = L.T^{-2}. \quad (1.38)$$

2/ The dimension of the pressure (P);

$$[P] = \frac{[F]}{[S]} = \frac{M.L.T^{-2}}{L^2} = M.L^{-1}.T^{-2}. \quad (1.39)$$

Exercise 4:

* The dimension of the viscosity (η):

From the law

$$\eta = \frac{2}{9} \rho_e (d-1) \frac{r^2 g}{v}. \quad (1.40)$$

The dimension of the viscosity reads

$$[\eta] = \frac{[\rho_e] [r]^2 [g]}{[v]}, \quad (1.41)$$

$$= \frac{\frac{M}{L^3} \times L^2 \times \frac{L}{T^2}}{\frac{L}{T}}, \quad (1.42)$$

$$= ML^{-1}T^{-1}. \quad (1.43)$$

where $[\rho_e] = \frac{M}{L^3}$, $[r] = L$, $[g] = \frac{L}{T^2}$ and $v = \frac{L}{T}$.

Exercise 5:

* The dimension of the diffusion coefficient D ;

From Fick's law (1.15), the dimension of the diffusion coefficient D is given by;

$$[D] = \frac{[x] [n]}{[S] [c] [t]}, \quad (1.44)$$

$$= \frac{LN}{L^2 N L^{-3} T}, \quad (1.45)$$

$$= L^2 T^{-1}, \quad (1.46)$$

where $[x] = L$, $[n] = N$, $[S] = L^2$, $[c] = NL^3$ and $[t] = T$.

Chapter 2

Optics

2.1 Introduction

In this chapter, our aim is to study geometric optics. We will begin by discussing the nature of light, followed by an exploration of the properties and principles of geometric optics. We will specifically focus on the phenomena of refraction and reflection on both planar and spherical surfaces. Subsequently, we will apply the governing laws to a range of simple optical systems, such as a plane-parallel transparent plate, prisms, and lenses. Finally, we will examine the human eye and its interaction with geometric optics, as well as common vision defects and their correction. Additionally, we will discuss optical instruments used in the field of biology, such as magnifying glasses and microscopes, and how they function based on the principles of geometric optics.

2.2 Nature of light

Light has two aspects:

Corpuscular (Newton): information is transported by grains of light, photons (massless particles).

Wave (Huughense, Fresnel): light is a wave characterized by an electric field and a magnetic field perpendicular to each other (electromagnetic wave).

2.3 Geometric Optics

Geometric optics is an **approximation** valid when the wavelength of light is significantly smaller than the dimensions of the objects it encounters. The light ray concept is a simplification : light exhibits both wave and particle properties.

2.3.1 Principles of geometric optics and propagation of light

The principles of geometric optics are:

1/ **Rectilinear Propagation:** In a **homogeneous, transparent, and isotropic** medium, light rays travel in **straight lines**.

2/ **Independence of Rays:** Light rays do not interact with each other; they propagate **independently**.

3/ **Snell's Law:** At the interface between two media, light rays obey **Snell's laws** of **refraction** and **reflection**.

2.3.2 Definitions

Transparent medium

A transparent medium allows light rays to propagate without significant scattering or absorption. This means the rays maintain their straight paths, which is essential for the basic principles of geometric optics to hold true.

Homogeneous medium

A medium is said to be homogeneous if its physical properties are the same at every point.

Isotropic medium

A medium is said to be isotropic if its physical properties are the same in every direction.

Light ray

A light ray is the trajectory actually followed by light.

Light beams

The set of light rays constitutes a light beam.

* It can be: parallel, convergent and divergent (See Figure 2.1).

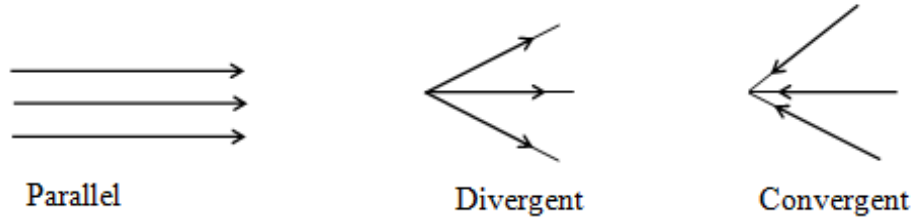


Figure 2.1: Light beams.

Fermat's principle

The path actually followed by light to go from one point to another is the one for which the travel time is minimum (in any extremum rule).

This principle has two immediate consequences;

- 1/ The principle of **rectilinear** propagation of light.
- 2/ The principle of **inverse return** of light.

Speed of light in a vacuum

Light propagates in a straight line at the speed c which is, by definition, the following:

$$c = 299792458 \text{ ms}^{-1} \simeq 3 \times 10^8 \text{ ms}^{-1}, \quad (2.1)$$

then, the **velocity** v of light in a **medium** depend to the refractive index n ;

$$n = \frac{c}{v}, \quad (2.2)$$

Her $v \leq c$, then $n \geq 1$. The refractive index n related to the **nature** of the medium;

Medium	Refractive index
Air	1
Water	1.33
Ethanol:	1.36
Glycerol	1.47
Benzene:	1.50

Real and virtual object or image

Real object Point A is a real object, a physical source from which light originates, and is therefore tangible (See Figure 2.2) .

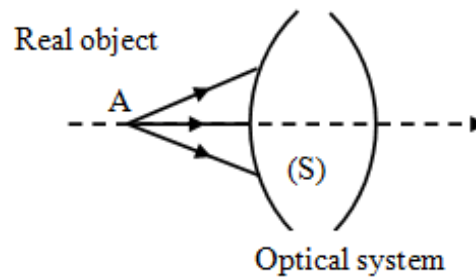


Figure 2.2: Real object.

Virtual object Point A is a virtual object, it represent the convergence point of a beam of converging rays, rays that are intercepted by the optical system before they reach that point (See Figure 2.3).

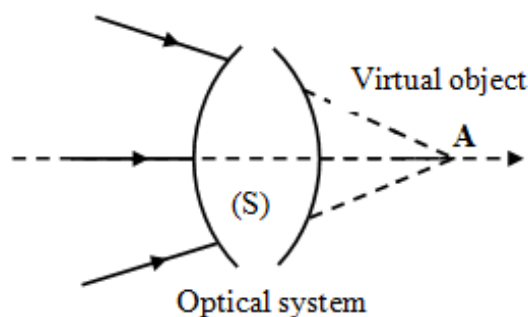


Figure 2.3: Virtual object.

Real image A real image is formed at point A' because the light rays physically converge at that location. This allows the image to be projected and viewed on a screen (See Figure 2.4).

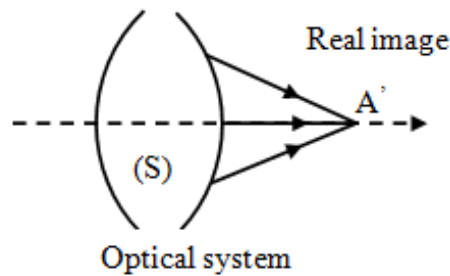


Figure 2.4: Real image

Virtual image A virtual image is formed at point A' , which is the point from which the diverging rays emerging from the optical system appear to originate (See Figure 2.5).

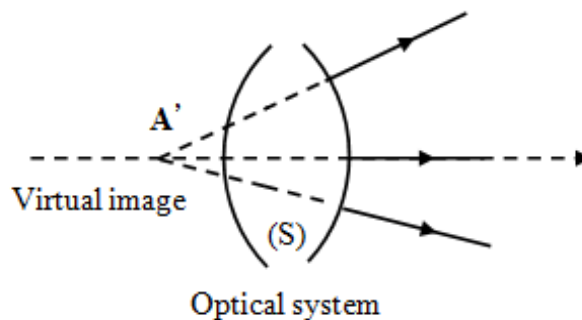


Figure 2.5: Virtual image.

Astigmatism and Gaussian Conditions

Gaussian Conditions We only consider centered optical systems, that is, systems for which there exists an axis of rotational symmetry called the optical axis. It is then shown that such an optical instrument will produce a good quality image of an object if the following two conditions, known as Gaussian conditions, are satisfied:

- The objects are of small extent and located in the vicinity of the optical axis.
- The incident light rays make a small angle with the optical axis.

Astigmatism A system (S) is **stigmatic** when to each point (A) of an object corresponds a point (A') of the image. We say that (A) and (A') are conjugate in (S) . In other words, all the rays originating from (A) that pass through (S) go through (A') . This is **absolute astigmatism**. We speak of **approximate astigmatism** when all the rays pass in the vicinity of (A') (See Figure 2.6).

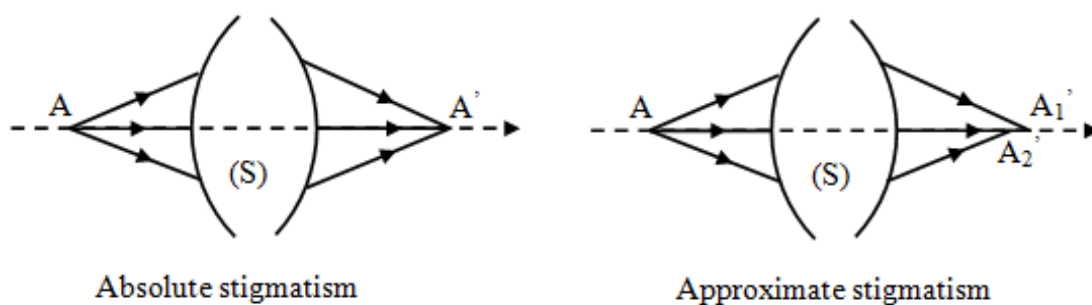


Figure 2.6: Absolute and approximate stigmatism.

Note

Under **Gaussian conditions**, the image of a plane object perpendicular to the optical axis is plane and perpendicular to the optical axis (**planetesimal**).

2.4 Refraction

Experiments have shown that the refraction of a light ray at the interface between two transparent media depends on their respective refractive indices, n_1 , n_2 and the geometry of the interface. Therefore, we will explore this phenomenon for both **planar** and **spherical** interfaces.

2.4.1 Refraction at a planar interface

Light refraction occurs when a light ray travels from a transparent medium with refractive index n_1 to another transparent medium with refractive index n_2 through a **planar interface**. We can analyze the refraction based on the relationship between n_1 and n_2 :

$$\left\{ \begin{array}{ll} n_1 < n_2, & \text{refraction occurs.} \\ n_1 > n_2, & \left\{ \begin{array}{ll} i \leq i_l, & \text{refraction occurs,} \\ i > i_l, & \text{total internal reflection occurs,} \end{array} \right. \end{array} \right. \quad (2.3)$$

where i_l is the critical angle. Then we have the following two cases:

Case1: Refraction of light from a **rarer** to a **denser** medium ($n_1 < n_2$) (Figure 2.7):

When a light ray travels from a **rarer** medium into a **denser** medium :

- The refracted light ray **bends towards** the normal.
- The angle of refraction r is **less** than the angle of incidence i : ($r < i$) .
- The relationship between the angle of incidence (i) and the angle of refraction (r)

is described by **Senell-Discart's** Law of refraction.

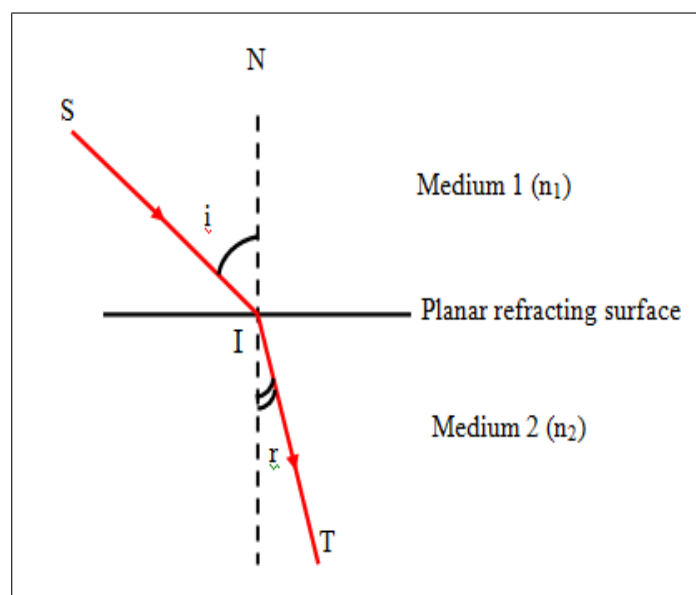


Figure 2.7: Refraction from a rarer medium to a denser medium.

Planar refracting Surface: An interface separating two transparent, homogeneous, and isotropic media with distinct refractive indices.

Normal (IN): The line perpendicular to the planar surface at the point of incidence (I).

Point of incidence (I): The point where the incident ray intersects the dioptric interface.

Incident ray (SI): The ray of light approaching the dioptric interface.

Refracted ray (IR): The ray of light that passes through the dioptric interface and into the second medium.

Plane of incidence: The plane defined by the incident ray and the normal to the dioptric interface at the point of incidence.

Angle of incidence (i): The angle between the **incident ray** and the **normal**.

Angle of Refraction (r): The angle between the **refracted ray** and the **normal**.

Note

In this context, the term "**planar refracting surface**" is employed in place of "**diopter**" as "dioptr" typically functions as a **unit** of optical power.

Senell-Discart's law of refraction:

1/ The incident ray, the normal to the refracting surface at the point of incidence, and the refracted ray all lie in the same plane, known as the plane of incidence or the plane of refraction.

2/ The angles of incidence (i) and refraction (r) are related by the following equation:

$$n_1 \sin i = n_2 \sin r, \quad (2.4)$$

Critical angle:

The critical angle (i_l) is the angle of incidence (i) in the denser medium (n_1) for which the angle of refraction (r) in the rarer medium (n_2) is 90° (See Figure 2.8).

It only occurs when light travels from a denser medium (n_1) to a rarer medium (n_2), where ($n_1 > n_2$).

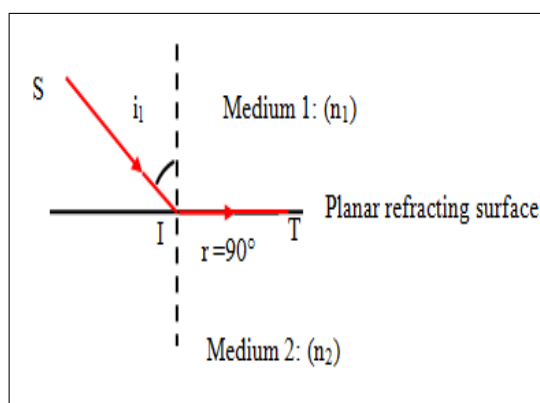


Figure 2.8: Critical angle.

The critical angle (i_l) is calculated using the following formula:

$$i_l = \text{Arcsin} \frac{n_1}{n_2}, \quad (2.5)$$

Case 2 : Refraction from a **denser** to a **rarer** medium ($n_1 > n_2$).

We distinguish two scenarios:

**** Refraction** ($i \leq i_l$) (See Figure 2.9):

- The light ray travels from a **denser** medium to a **rarer** medium.
- The refracted ray **bends away** from the normal.
- The angle of refraction (r) is **greater** than the angle of incidence (i): ($r > i$).

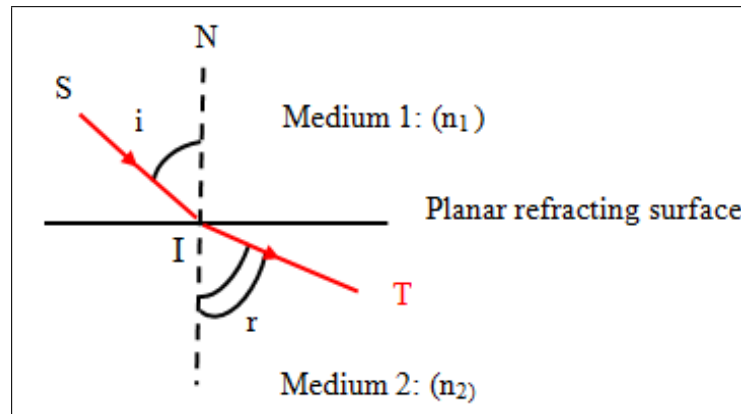


Figure 2.9: Refraction from a denser medium to a rarer medium.

**** Total internal reflection** ($i > i_l$) (See Figure 2.10):

- The incident light ray is **reflected** back into the denser medium.
- The relationship between the angle of incident and reflected ray is governed by **Senell-Discart's** law of reflection (angle of incident = angle of reflection).

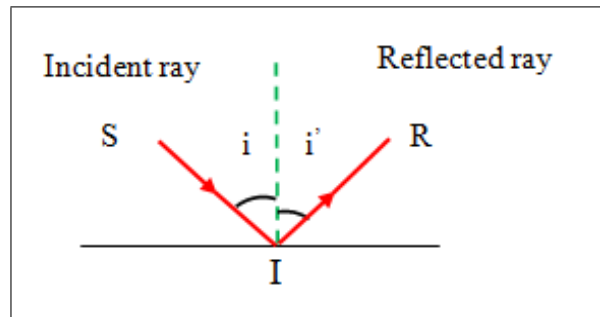


Figure 2.10: Total internal reflection.

Senell-Discart's law of reflection:

1/ The incident ray, the reflected ray, and the normal to the reflecting surface at the point of incidence all lie in the same plane, known as the plane of incidence (or plane of reflection).

2/ The angle of reflection (i') is **equal** to the angle of incidence (i):

$$i = i', \quad (2.6)$$

Normal incidence: When the angle of incidence (i) is 0° , the angle of refraction (r) is also 0° (See Figure 2.11).

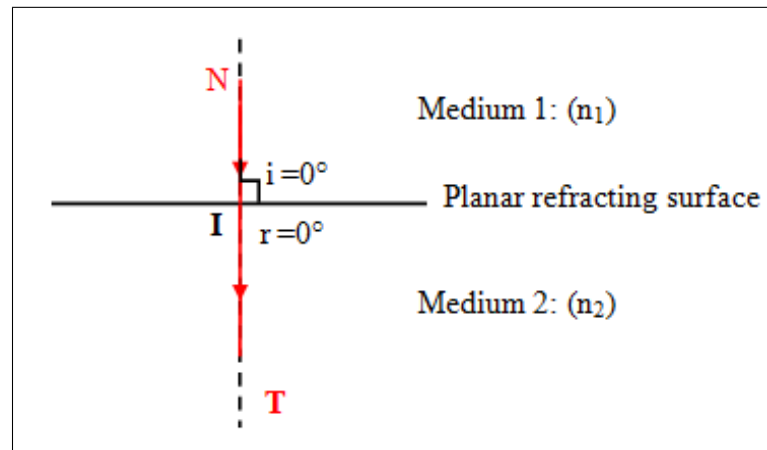


Figure 2.11: Normal incidence.

Image formation by refraction at a planar interface

To determine the image A' (observed in medium 1 with refractive index n_1) of an object point A located in medium 2 with refractive index n_2 , we follow the steps:

- We trace two incident rays originating from point A and passing from medium 2 to medium 1.
- The image A' is formed at the intersection of the refracted rays (or their extensions).

We consider two scenarios:

* **Case 1** : $n_1 > n_2$ (The image A' **approaches** the observer (See Figure 2.12)):

- The first ray is perpendicular to the interface between the two mediums, so it does not deviate (does not refract).
- A second ray, traveling from the **denser** medium (Medium 2) to the **rarer** medium (medium 1), **bends away** from the normal.

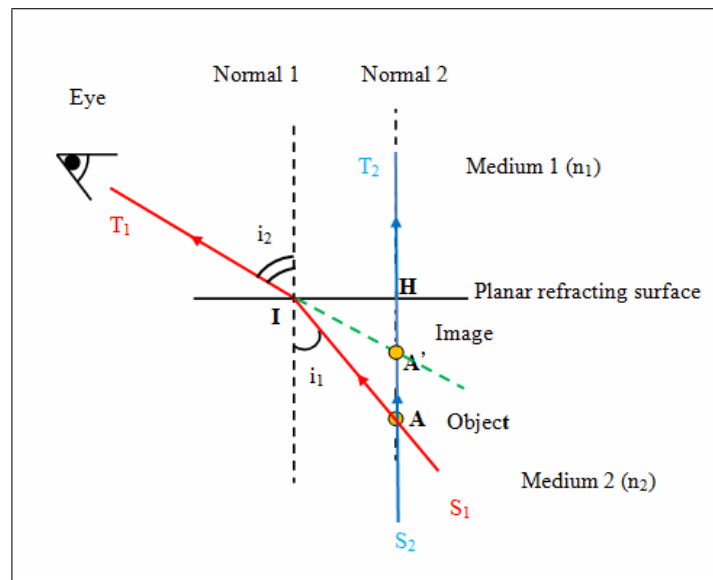


Figure 2.12: Image formation by refraction at a planar interface (Denser medium to rarer medium).

* **Case 2:** $n_1 < n_2$ (Image appears **farther** from the observer (See Figure 2.13))

- A ray incident normally (perpendicular) to the interface passes through without deviation (no refraction).
- A second ray, traveling from the **rarer** medium (Medium 2) to the **denser** medium (medium 1), **bends toward** the normal.

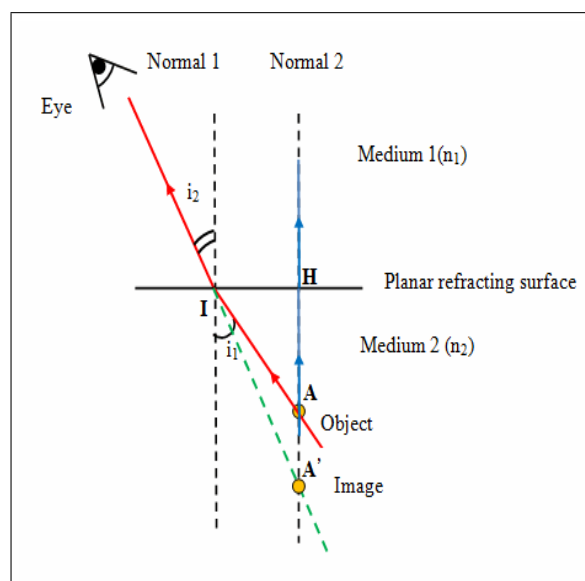


Figure 2.13: Image formation by refraction at a planar interface (Rarer medium to denser medium).

Conjugation formula

The conjugation formula for a **planar refracting surface** relates the object and image distances:

$$\frac{\overline{HA'}}{\overline{HA}} = \frac{n_2}{n_1}. \quad (2.7)$$

2.4.2 Parallel slab:

A parallel slab is a homogeneous transparent medium bounded by two parallel flat surfaces.

* **Case1:** Slab Between two **different** media ($n_1 \neq n_2$)

When a parallel slab (refractive index n) separates two media with different refractive indices (n_1 and n_2), the **emergent** ray is **deviated** (See Figure 2.14).

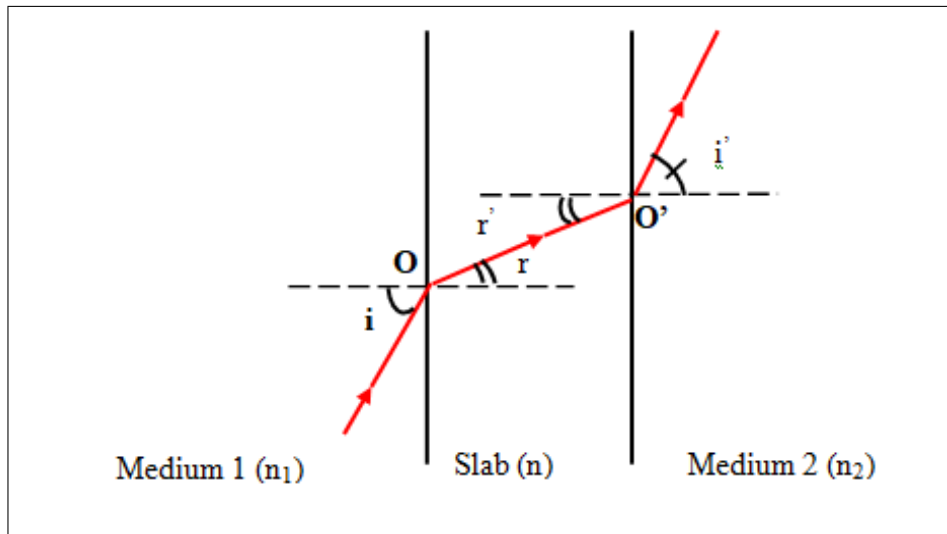


Figure 2.14: Parallel slab separates two media with different refractive indices.

Applying Snell's law at each interface:

Medium 1 to slab;

$$n_1 \sin i = n \sin r, \quad (2.8)$$

Slab to medium 2;

$$n \sin r' = n_2 \sin i', \quad (2.9)$$

Since the slab's surfaces are parallel, the angles of refraction and incidence within the slab are equal:

$$r = r', \quad (2.10)$$

Combining the equations (2.8),(2.9) and (2.10), we get:

$$n_1 \sin i = n_2 \sin i', \quad (2.11)$$

If $n_1 \neq n_2$, then

$$i \neq i', \quad (2.12)$$

indicating that the emergent ray is **deviated**.

* **Case2:** Slab immersed in a **single** medium ($n_1 = n_2$)

When a parallel slab (refractive index n) is immersed in a single medium (refractive index n_1), the emergent ray is only laterally displaced (**translated**) (Figure 2.15).

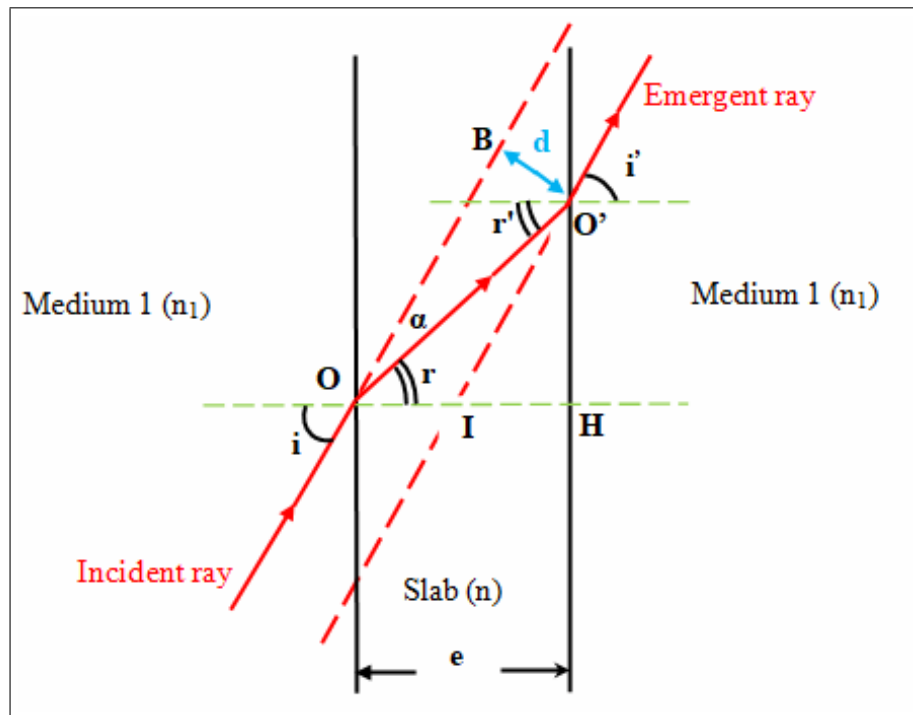


Figure 2.15: Parallel slab is immersed in a single medium.

Lateral displacement

Let e be the thickness of the slab.

Let $\alpha = i - r$ be the difference between the incident and refracted angles.

In **right triangle** $OO'B$, where d is the lateral displacement:

$$\sin \alpha = \frac{d}{OO'}, \quad (2.13)$$

In **right triangle** $OO'H$;

$$\cos \alpha = \frac{e}{OO'}, \quad (2.14)$$

Combining these equations (2.13) and (2.14), we get

$$d = e \frac{\sin(\alpha)}{\cos(r)} = e \frac{\sin(i - r)}{\cos(r)}. \quad (2.15)$$

* **Grazing incidence** ($i = 90^\circ$)(See Figure 2.16):

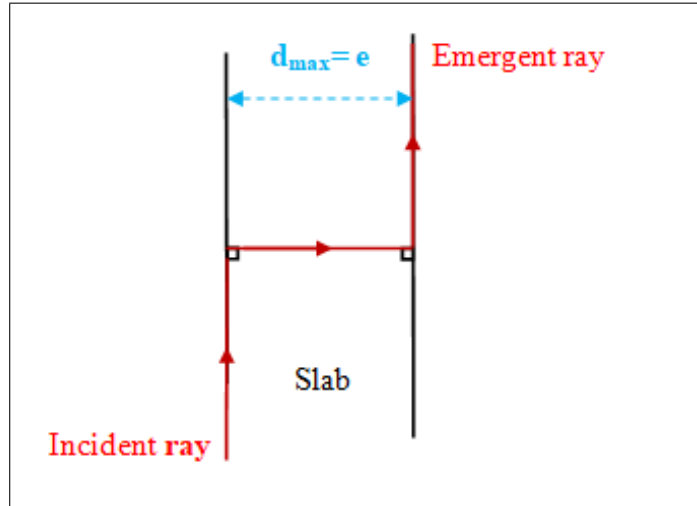


Figure 2.16: Parallel slab with grazing incidence.

The **maximum** lateral displacement occurs:

$$d_{\max} = e \frac{\sin(90 - r)}{\cos(r)} = e, \quad (2.16)$$

Thus

$$d_{\max} = e. \quad (2.17)$$

Where $\sin(90 - r) = \cos(r)$.

** **Normal incidence** ($i = 0^\circ$) (Figure 2.17):

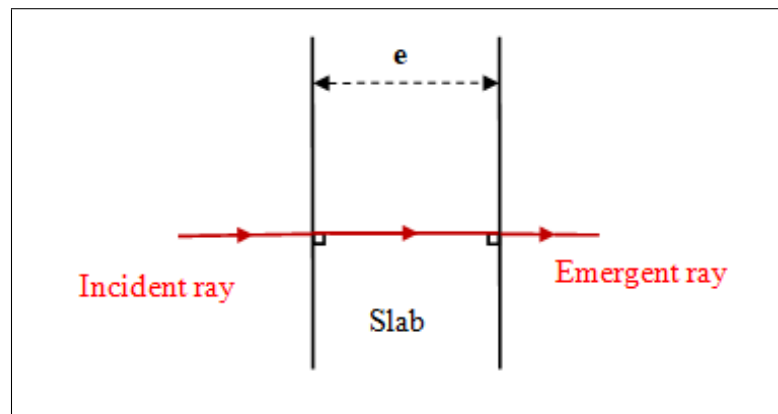


Figure 2.17: Parallel slab with normal incidence.

The lateral displacement is zero;

$$d = e \frac{\sin(0 - r)}{\cos(r)} = 0. \quad (2.18)$$

2.4.3 Prism

In optics, a prism is a transparent medium with two **non-parallel** planar faces. It is typically considered to be made of a material with refractive index n and is surrounded by air ($n_{air} = 1$) (Figure 2.18).

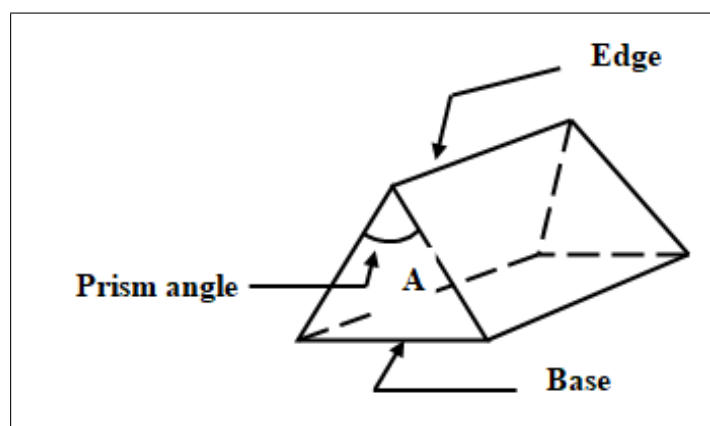


Figure 2.18: Prism.

Path of a light ray in the prism (Figure 2.19):

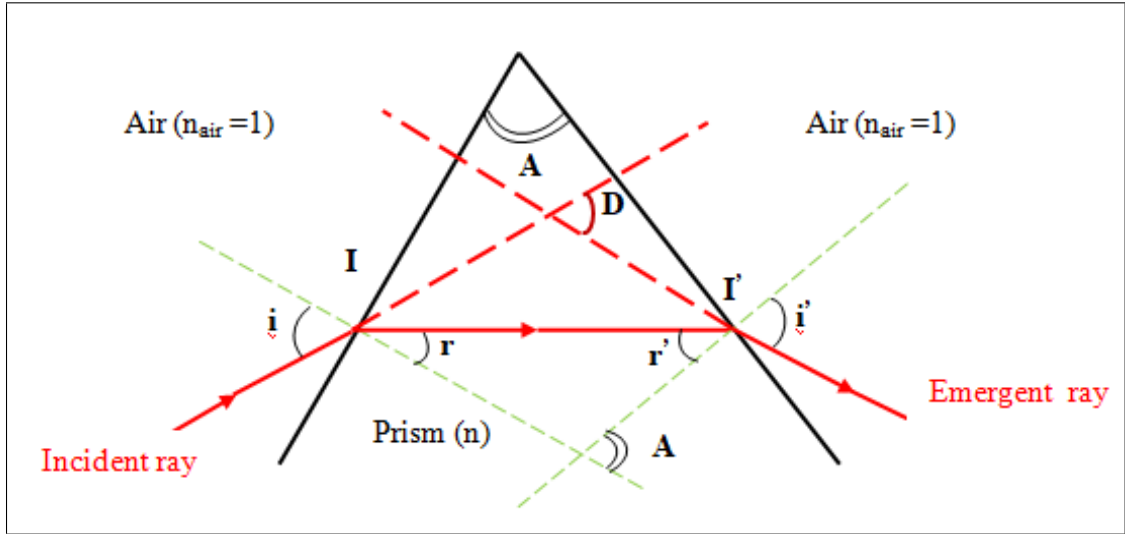


Figure 2.19: Path of a light ray in the prism.

The four fundamental formulas of the prism:

$$\sin i = n \sin r, \quad (2.19)$$

$$\sin i' = n \sin r', \quad (2.20)$$

$$A = r + r', \quad (2.21)$$

$$D = i + i' - A. \quad (2.22)$$

where

A : Angle of the prism.

i : Angle of incidence.

i' : Angle of emergence.

r, r' : Angles of refraction.

D : Angle of deviation.

Emergency conditions:

For a light ray incident on a prism to emerge from the opposite face, the following necessary and sufficient condition must be satisfied:

$$r' < r_l, \quad (2.23)$$

where

r' is the angle of refraction at the **second** surface of the prism.

r_l is the **critical angle** for total internal reflection at the prism-air interface, defined by:

$$r_l = \text{Arc sin } \frac{1}{n}, \quad (2.24)$$

Alternatively, the condition for emergence can be expressed in terms of the angle of incidence at the first surface (i):

$$i \geq i_0, \quad (2.25)$$

where (i_0) is the **minimum** angle of incidence required for **emergence**, given by:

$$i_0 = \text{Arc sin } [n \sin(A - r_l)], \quad (2.26)$$

* A necessary (but not sufficient) condition for the emergence of the ray is:

$$A < 2r_l. \quad (2.27)$$

Minimum value of deviation angle (D_m)

The **minimum** deviation angle (D_m) occurs when the angle of incidence (i) is **equal** to the angle of emergence (i') and consequently, the angle of refraction at the first surface (r) is **equal** to the angle of incidence at the second surface (r'). Under these conditions:

$$i = i' = i_m, \quad r = \frac{A}{2}, \quad (2.28)$$

The minimum deviation angle can then be calculated using:

$$D_m = 2i_m - A, \quad (2.29)$$

Alternatively, D_m can be found by solving the following equation:

$$n \sin \frac{A}{2} = \sin \left(\frac{A + D_m}{2} \right). \quad (2.30)$$

2.5 Refraction at a spherical surface

A spherical refracting surface is a portion of a sphere that separates two media with different refractive indices, denoted as n_1 and n_2 .

A spherical refracting surface is characterized by its **center** of curvature (C) and its **radius** of curvature (R). The radius of curvature is the distance from the **vertex** (S) of the surface to the center of curvature :

$$R = \bar{SC}. \quad (2.31)$$

There are two types of spherical refracting surfaces, classified by the **sign** of the **radius** of curvature (Figure 2.20):

Concave surface: The center of curvature (C) lies on the same side as the outgoing refracted light.

By convention, the radius of curvature for a concave surface is considered negative ($R < 0$).

Convex surface: The center of curvature (C) lies on the opposite side from the outgoing refracted light.

By convention, the radius of curvature for a convex surface is considered positive ($R > 0$).

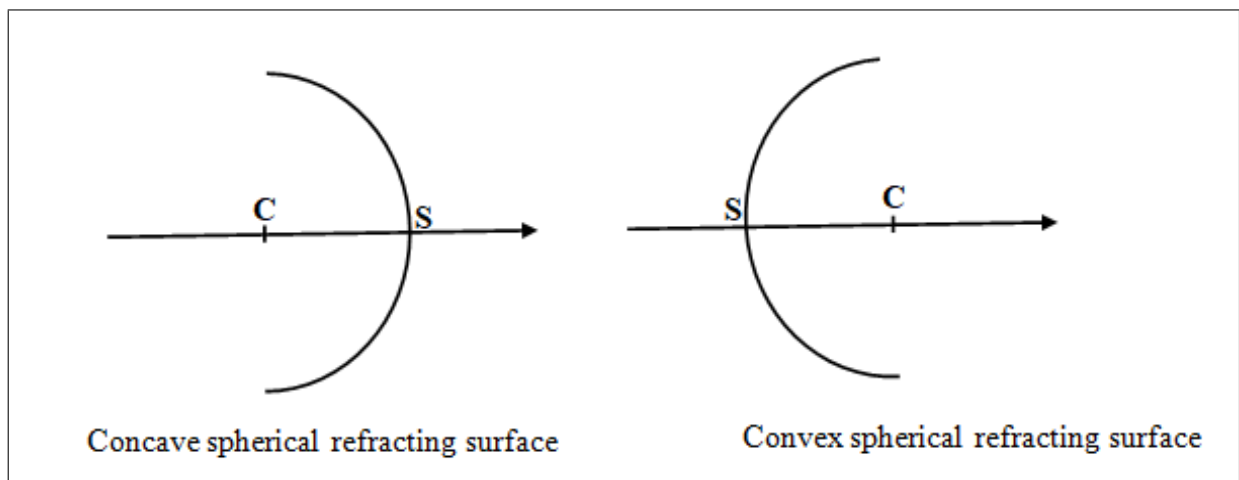


Figure 2.20: Types of spherical refracting surfaces.

Convention of signs

1/ Light rays are assumed to travel from **left to right**, incident on the vertex S .

2/ All distances are measured from the vertex S .

3/ Distances are considered **positive** if measured to the **right** of the vertex (S) along the optical axis, and **negative** if measured to the **left** of the vertex (S) along the optical axis.

Conjugate relation (Gaussian formula)

The conjugate relation of the spherical refracting surface is given by

$$\frac{n_2}{\overline{SA'}} - \frac{n_1}{\overline{SA}} = \frac{n_2 - n_1}{\overline{SC}}, \quad (2.32)$$

where :

$\overline{SA}$: Object distance.

$\overline{SA'}$: Image distance.

$\overline{SC}$: Radius of curvature.

Vergence (V):

The vergence (V) of a spherical refracting surface is a measure of its power to converge or diverge light. It is defined by the relationship:

$$V = \frac{n_2 - n_1}{\overline{SC}}, \quad (2.33)$$

Nature of the spherical refracting surface

The nature of the spherical refracting surface (whether it is converging or diverging) is determined by the **sign** of the vergence (V):

- If $V > 0$: The spherical refracting surface is **converging**. It causes parallel rays of light to converge towards a focal point.
- If $V < 0$: The spherical refracting surface is **diverging**. It causes parallel rays of light to diverge, appearing to originate from a focal point.

Unit of Vergence

Vergence is measured in diopters (δ). ($1\delta = 1m^{-1}$)

Object focus, image focus and focal length

Object focus (F) (or primary focal point): This is an axial point such that any ray emanating from it (or appearing to converge towards it) will travel **parallel** to the principal axis after refraction by the spherical surface.

Image focus (F') (or secondary focal points): This is an axial point such that any incident ray traveling **parallel** to the principal axis will, after refraction, either converge to this point (for a real image focus) or appear to diverge from it (for a virtual image focus).

Primary focal length (f): The distance between the vertex (S) of the refracting spherical surface and the object focus (F). It is given by:

$$f = \bar{SF} = -\frac{n_1}{V} = -\frac{n_1}{n_2 - n_1} \bar{SC}, \quad (2.34)$$

Secondary focal length (f'): The distance between the vertex (S) of the refracting spherical surface and the image focus (F'). It is given by:

$$f' = \bar{SF}' = \frac{n_2}{V} = \frac{n_2}{n_2 - n_1} \bar{SC}, \quad (2.35)$$

Notes Here's a breakdown for concave and convex surfaces:

* **Concave spherical refracting surface ($R < 0$):**

Case 1: ($n_1 < n_2$)

Light travels from a less dense to a denser medium:

- Object focus (F) is real ($SF > 0$).
- Image focus (F') is virtual ($SF' < 0$).
- Vergence (V) > 0 (Convergent surface).

Case 2: $n_1 > n_2$

Light travels from a denser to a less dense medium:

- Object focus (F) is virtual ($SF < 0$).
- Image focus (F') is real ($SF' > 0$).
- Vergence (V) < 0 (Divergent surface).

** **Convex Spherical Refracting Surface ($R > 0$):**

Case 1: ($n_1 < n_2$)

Light travels from a less dense to a denser medium:

- Object focus (F) is virtual ($SF < 0$).
- Image focus (F') is real ($SF' > 0$).
- Vergence (V) > 0 (Convergent surface).

Case 2: $n_1 > n_2$

Light travels from a denser to a less dense medium:

- Object focus (F) is real ($SF > 0$).
- Image focus (F') is virtual ($SF' < 0$).
- Vergence (V) < 0 (Divergent surface).

Magnification of a spherical refracting surface

Nature of the image based on magnification (γ):

The **sign** and **magnitude** of the **magnification** (γ) provide information about the nature of the image:

inverted, enlarged and reduced.

The magnification (γ) of a spherical refracting surface describes the ratio of the image height ($\overline{A'B'}$) to the object height ($\overline{AB}$).

It indicates how much the image is **enlarged** or **reduced** and whether it is **upright** or **inverted** relative to the object.

The magnification (γ) is defined as:

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{n_1}{n_2} \frac{\overline{SA'}}{\overline{SA}}, \quad (2.36)$$

Nature of the image

Nature of the image based on magnification (γ):

*** Sign of Magnification:**

$\gamma > 0$ (Positive Magnification): The image is **upright (erect)** relative to the object. This means the image has the same orientation as the object.

$\gamma < 0$ (Negative Magnification): The image is **inverted** relative to the object. This means the image is flipped upside down compared to the object.

* **Magnitude of Magnification ($|\gamma|$):**

$|\gamma| > 1$: The image is **enlarged (magnified)** compared to the object. The image appears larger than the object.

$|\gamma| < 1$: The image is **reduced (diminished)** compared to the object. The image appears smaller than the object.

$|\gamma| = 1$: The image has the **same size** as the object.

Geometric construction of the image

Image of material point To draw the image (B') of a material point (B), we can trace the paths of three particular rays originating from (B) whose behavior after refraction is known:

1/ A ray of light (R_1) incident at the center (C) of the spherical refracting surface, passes undeviated (R'_1) through convex and concave spherical refracting surface.

2/ A ray of light incident (R_2) parallel to the principal axis of the spherical refracting surface, after refraction (R'_2) passes through the image focus (F') (in a converging spherical refracting surface) or appears to come from the image focus (F') (in a diverging spherical refracting surface).

3/ A ray of light (R_3) passing through the object focus (F), (in a converging spherical refracting surface) or directed towards the object focus (F), (in a diverging spherical refracting surface), emerges parallel to the principal axis after refraction (R'_3).

4/ The image (B') of the material point (B) is formed at the point where the three refracted rays (R'_1 , R'_2 and R'_3) intersect (for a real image) or appear to diverge from (for a virtual image).

Image of an object extended perpendicular to the optical axis: In order to construct

the image of the object AB perpendicular to the optical axis, we follow the steps:

1/ Drawing the image B' of point B (as described for a material point).

2/ The image A' of point A is the projection of image B' on the optical axis.

3/ The image of object AB is $A'B'$.

Examples:

* **Concave refracting surface:**

1/ Converging spherical refracting surface (See Figure 2.21):

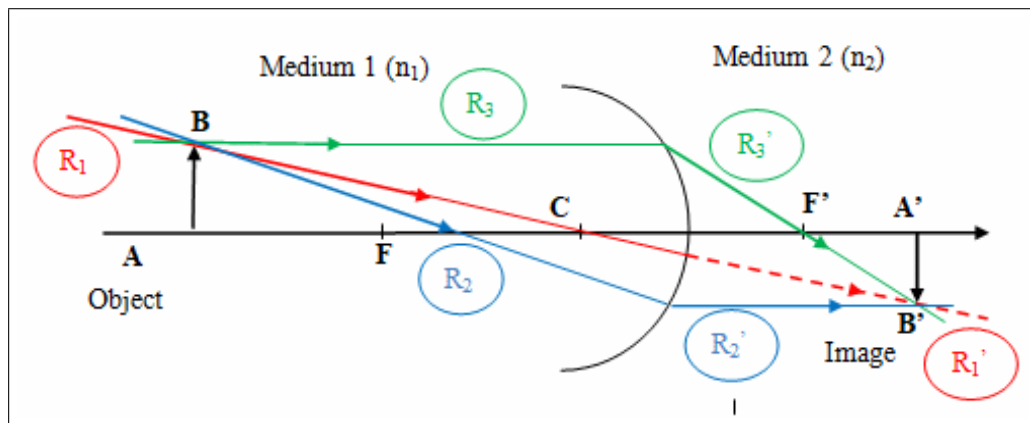


Figure 2.21: Concave refracting surface (Converging surface)

2/ Diverging refracting surface (Figure 2.22):

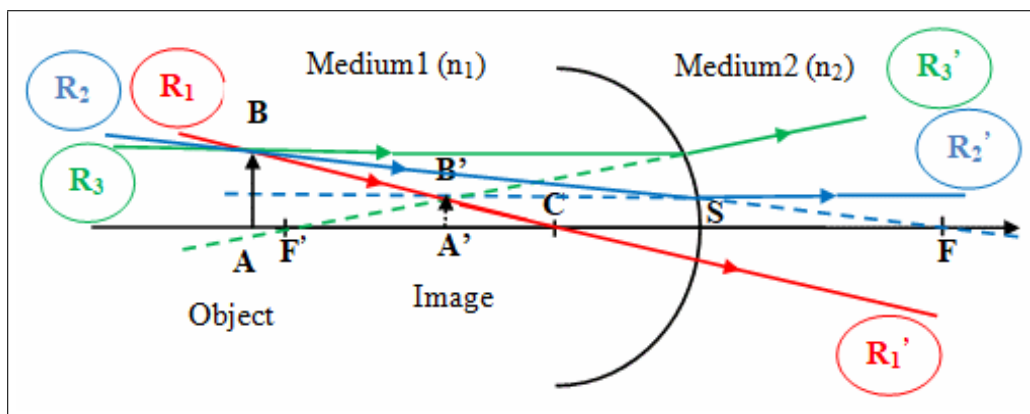


Figure 2.22: Concave refracting surface (Diverging surface).

** Convex refracting surface:

1/ Converging spherical refracting surface (Figure 2.23);

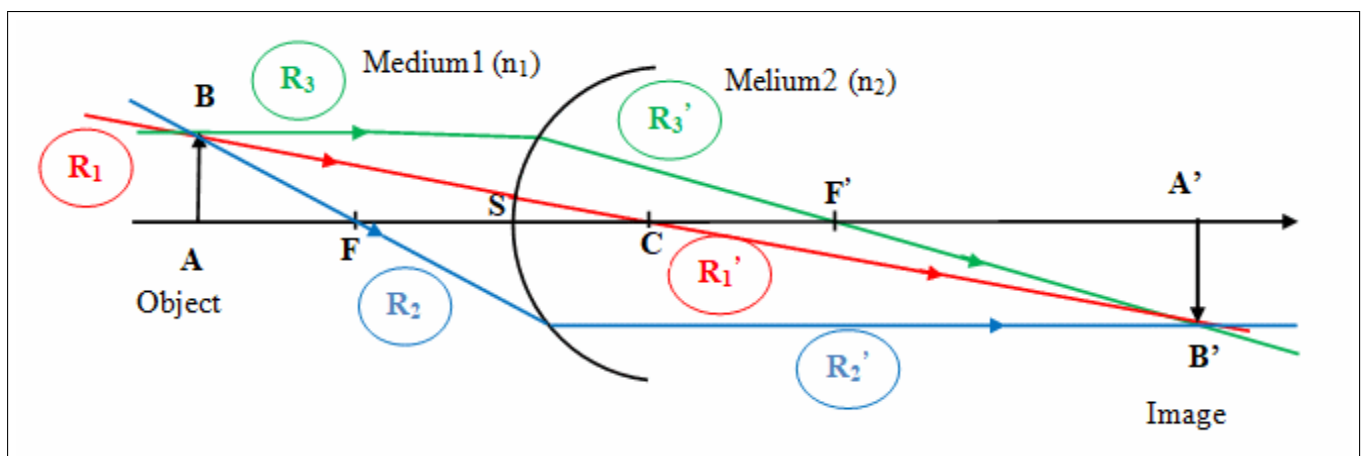


Figure 2.23: Convex refracting surface (Converging surface).

2/ Diverging spherical refracting surface (See Figure 2.24);

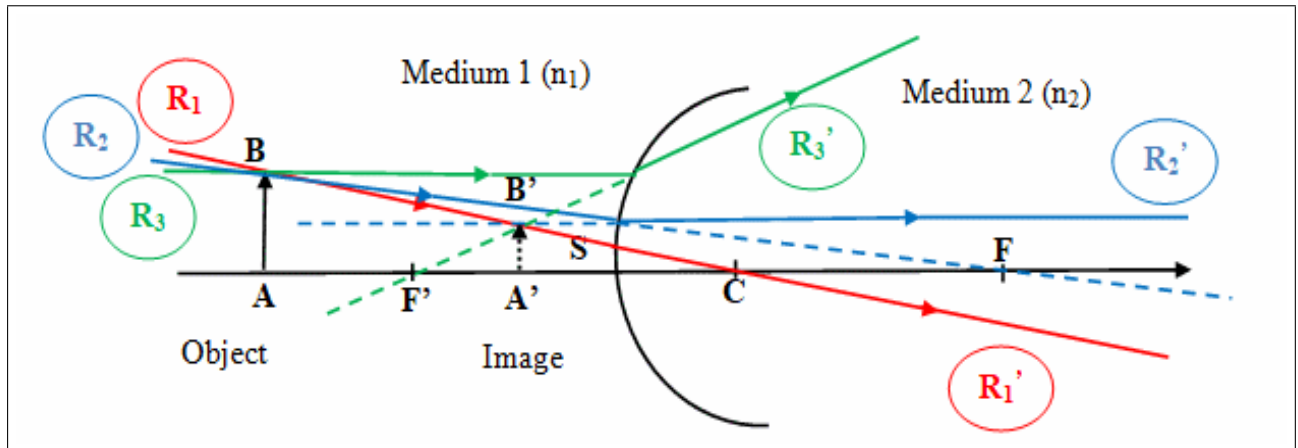


Figure 2.24: Convex refracting surface (Diverging surface).

Note:

The object can be moved to other positions on the optical axis in order to draw images of the selected positions.

The nature of the object or the formed image: The nature of the object and the formed image (whether real or imaginary) can be determined by the sign of their positions relative to the refracting surface:

*** Object**

Real Object: If the object is located **before** the spherical refracting surface, its position ($\bar{SA}$) is considered negative ($\bar{SA} < 0$).

Imaginary Object: If the object is located **after** the spherical refracting surface, its position ($\bar{SA}$) is considered positive ($\bar{SA} > 0$).

**** Image**

Real Image: If the image is formed after the spherical refracting surface, its position ($\bar{SA}'$) is considered positive ($\bar{SA}' > 0$).

Imaginary Image: If the image is formed before the spherical refracting surface, its position ($\bar{SA}'$) is considered negative ($\bar{SA}' < 0$).

2.5.1 Thin lenses

A lens is defined as a transparent medium with a refractive index greater than one ($n > 1$), bounded primarily by two spherical refracting surfaces or by one plane refracting surface and one spherical refracting surface (Figure 2.25) .

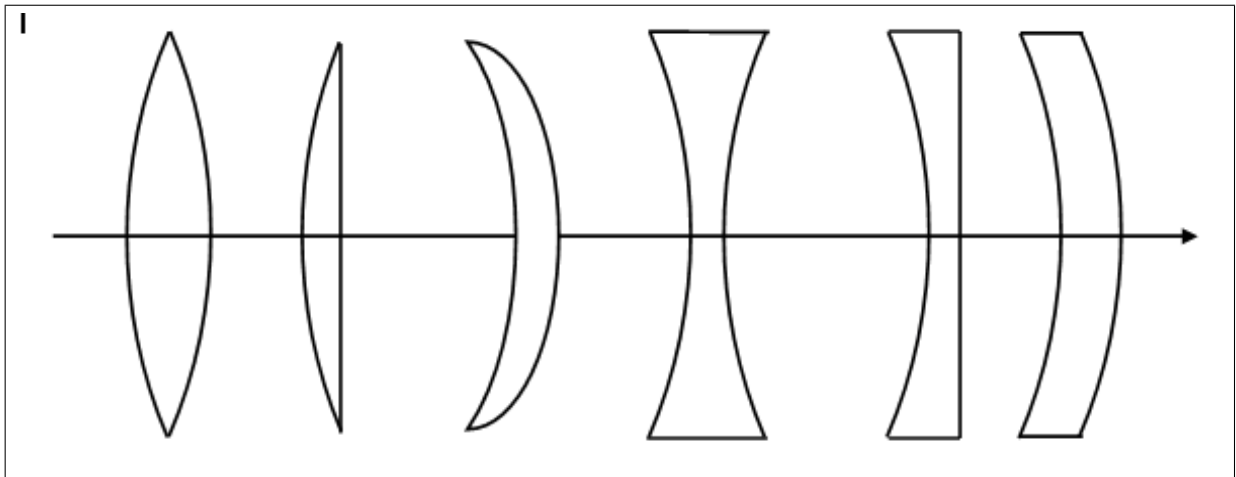


Figure 2.25: Thin lenses.

Thin Lens Approximation:

A lens is considered "**thin**" when its thickness measured along the optical axis (axial thickness) between its vertices S_1 and S_2 is negligibly small compared to the magnitudes of its two radii of curvature, $|R_1|$ and $|R_2|$, and also compared to the difference between these radii (Figure 2.26).

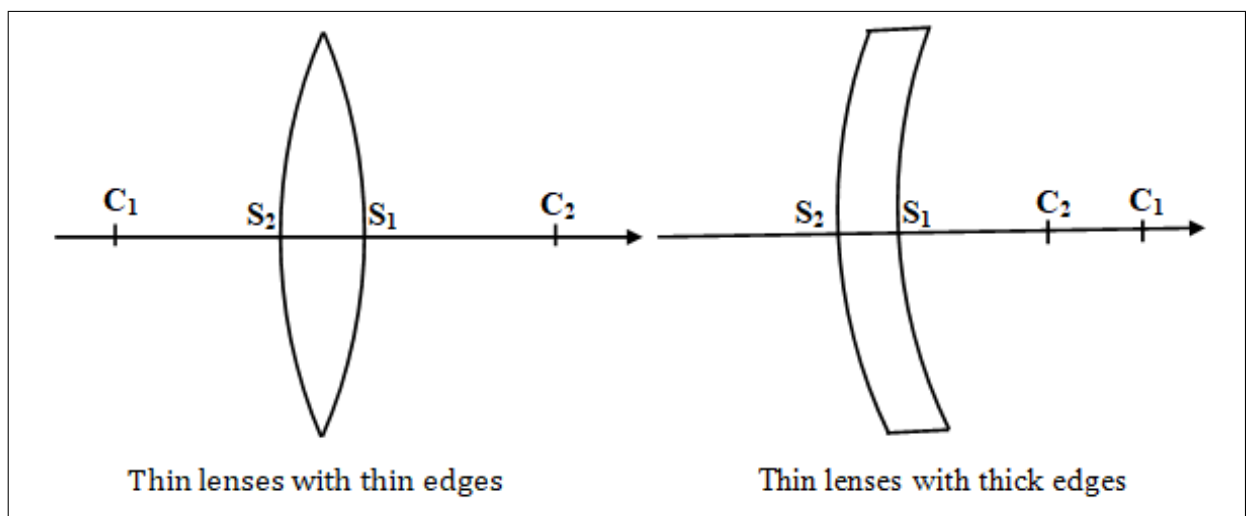


Figure 2.26: Thin Lens Approximation.

Graphic representations of the lenses

We have the two graphics representations of the lenses (Figure 2.27);

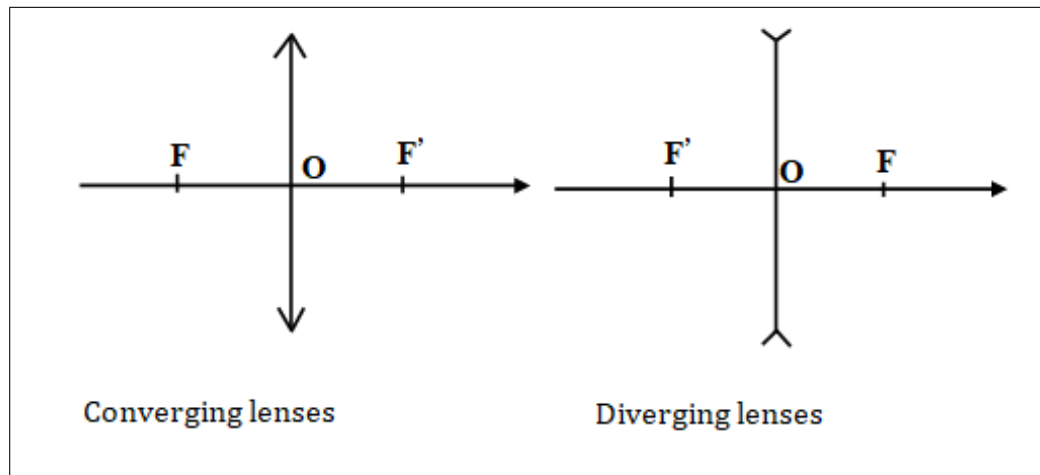


Figure 2.27: Graphic representations of the lenses.

with O : optical center and F , F' are object and image focuses of the lens.

Object focal point, image focal point and focal length

Object focal point (primary focal point) (F): is an axial point having the property that any ray coming from it or proceeding toward it, travels parallel to the axis after refraction.

Image focal point (secondary focal point) (F'): is an axial point having the property that any incident ray travelling parallel to the axis will after refraction proceed toward or appear to come from F' .

Notes;

Symmetry: The two focal points (object focal point F and image focal point F') of a thin lens are located symmetrically with respect to the optical center (O) of the lens.

Converging Lens: For a converging lens, both the object focal point (F) and the image focal point (F') are real.

Diverging Lens: For a diverging lens, both the object focal point (F) and the image focal point (F') are virtual.

Focal length

The focal length (f') of a thin lens is defined as the distance between the optical center (O) of the lens and either of its focal points (F'). Mathematically:

$$f' = \overline{OF'}, \quad (2.37)$$

we always have

$$f' = -f, \quad (2.38)$$

where $f = \overline{OF}$.

Nature of a lens

The **nature** of a thin lens is determined by the **sign** of its **focal length**:

* A lens with a **positive** focal length ($f' > 0$) is a **converging** lens.

** A lens with a **negative** focal length ($f' < 0$) is a **diverging** lens.

Geometric construction:

Image of material point To draw the image (B') of a material point (B), we can trace the paths of three particular rays originating from (B) whose behavior after refraction is known:

1/ A ray of light (R_1) incident at the center (O) of the lens, passes undeviated (R'_1) through converging and diverging lenses.

2/ A ray of light incident (R_2) parallel to the principal axis of the lens, after refraction (R'_2) passes through the image focus (F') (in a converging lens) or appears to come from the image focus (F') (In a diverging lens).

3/ A ray of light (R_3) passing through the object focus (F), (in a converging lens) or directed towards the object focus (F), (In a diverging lens), emerges parallel to the principal axis after refraction (R'_3).

4/ The image (B') of the material point (B) is formed at the point where the three refracted rays (R'_1 , R'_2 and R'_3) intersect (for a real image) or appear to diverge from (for a virtual image).

Image of an object extended perpendicular to the optical axis: In order to construct

the image of the object AB perpendicular to the optical axis, we follow the steps:

- 1/ Drawing the image B' of point B (as described for a material point).
- 2/ The image A' of point A is the projection of image B' on the optical axis.
- 3/ The image of object AB is $A'B'$.

Example: 1/ Converging lens (Figure 2.28);

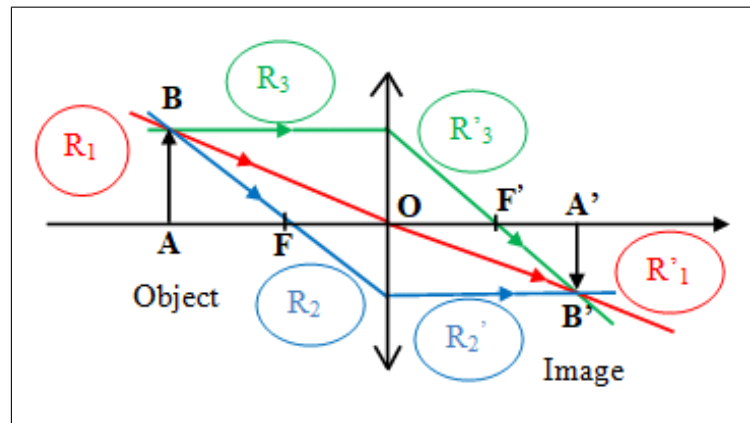


Figure 2.28: Geometric construction (Converging lens).

2/ Diverging lens (Figure 2.29);

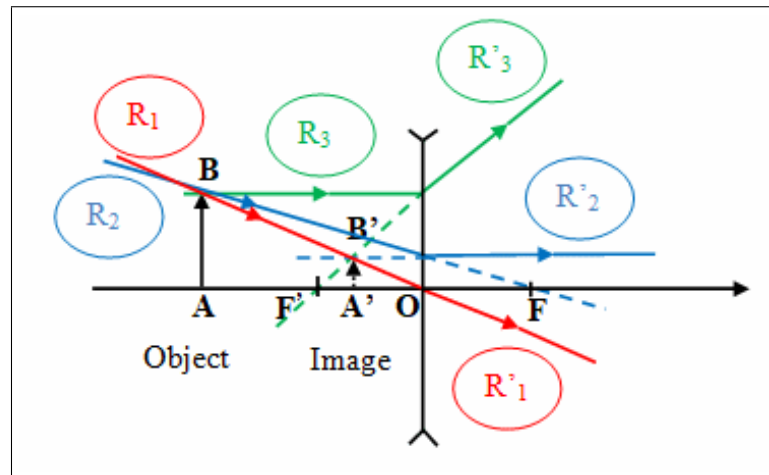


Figure 2.29: Geometric construction (Diverging lens).

Conjugation relation (Gaussian Lens Formula);

Consider a thin lens formed by two spherical refracting surfaces with radii of curvature R_1 and R_2 . An object (AB) is placed in front of the lens. The first spherical surface refracts light

from the object (AB) to form an intermediate image (A_1B_1). This intermediate image then acts as the object for the second spherical refracting surface, which further refracts the light to form the final image ($A'B'$). We assume the lens is thin, meaning the distance between the two vertices of the spherical surfaces (S_1 and S_2) is negligible and can be considered coincident at a single point O . We define the distances from this common vertex O as follows:

$$p = \bar{OA}, \quad p' = \bar{OA'}, \quad p = \bar{OA}. \quad (2.39)$$

We write the conjugate relation for each spherical refracting surface;

Spherical refracting surface 1:

$$\frac{n}{p_1} - \frac{1}{p} = \frac{n-1}{R_1}, \quad (2.40)$$

Spherical refracting surface 2:

$$\frac{1}{p'} - \frac{1}{p_1} = \frac{1-n}{R_2}, \quad (2.41)$$

From equations (2.40) and (2.41) we find the conjugate relation of a lens:

$$\frac{1}{p'} - \frac{1}{p} = \frac{1}{f'}, \quad (2.42)$$

with

$$f' = \frac{1}{(n-1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)}, \quad (2.43)$$

is the focal length of the lens.

Conjugate relation of a lens:

The conjugate relation of a thin converging or diverging lens given by;

$$\frac{1}{p'} - \frac{1}{p} = \frac{1}{f'}, \quad (2.44)$$

Magnification of a thin lens:

The magnification γ of a thin lens is defined as the ratio of the height of the image ($A'B'$) to the height of the object (AB). It is also related to the image distance (p') and the object distance (p):

$$\gamma = \frac{\overline{A'B'}}{\overline{AB}} = \frac{p'}{p}, \quad (2.45)$$

Nature of the image

The characteristics of the image formed by a thin lens can be determined by the sign and magnitude of the magnification (γ) and the sign of the image distance (p'):

*** Image Orientation:**

- $\gamma > 0$: The image is **upright** with respect to the object.
- $\gamma < 0$: The image is **inverted** with respect to the object.

**** Image Size:**

- $|\gamma| > 1$: The image is **enlarged (magnified)** compared to the object.
- $|\gamma| < 1$: The image is **reduced (diminished)** compared to the object.
- $|\gamma| = 1$: The image is the **same size** as the object.

***** Image Type:**

- $p' > 0$: The image is **real**.
- $p' < 0$: The image is **virtual**.

Vergence of a lens:

The vergence (or optical power) V of a thin lens quantifies its ability to converge or diverge light. It is defined as the reciprocal of the focal length (f'):

$$V = \frac{1}{f'}, \quad (2.46)$$

where

- V is the vergence in **diopters** (δ).
- f' is the focal length in **meters** (m).

Nature of the Lens

The **sign** of the vergence directly indicates the **nature** of the lens:

- $V > 0$: The focal length (f') is **positive**, indicating a **converging** lens.

- $V < 0$: The focal length (f') is **negative**, indicating a **diverging** lens.

2.6 Reflection

2.6.1 Plane mirror

A plane mirror is defined as a flat, smooth reflecting surface.

laws of reflection (Snell-Descartes laws)

1/ Coplanarity: The incident ray, the reflected ray, and the normal (a line perpendicular to the reflecting surface at the point of incidence) all lie in the same plane. This plane is known as the plane of incidence (which is also the plane of reflection).

2/ Equality of angles: The angle of reflection (i') is equal to the angle of incidence (i). These angles are measured between the normal and the respective rays (Figure 2.30):

$$i = i', \quad (2.47)$$

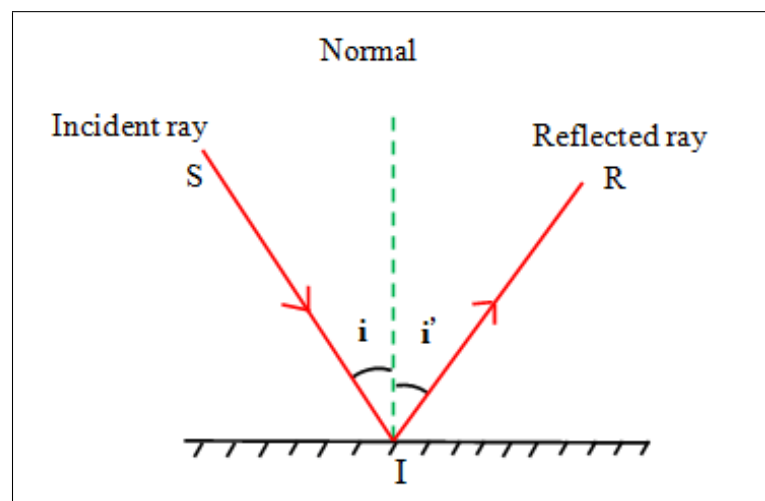


Image of a point object

To locate the image (A') of a point object (A) formed by a plane mirror, we can use the following steps based on the principles of reflection (See Figure 2.31):

1/ Draw at least two light rays (R_1 and R_2) originating from the point object A and striking the surface of the plane mirror at two distinct points, labeled I_1 and I_2 respectively.

2/ Draw the reflected rays (R'_1 and R'_2) at each point of incidence (I_1 and I_2) such that the angle between each reflected ray and its corresponding normal is equal to the angle between the incident ray and the same normal.

3/ The reflected rays (R'_1 and R'_2) are diverging after reflection. They do not physically intersect on the side where the reflected light travels. However, if you extend these reflected rays backward (mentally, using dashed lines) behind the mirror, they will appear to converge and intersect at a single point A' . This point A' is the virtual image of the point object A .

Note

The location of the virtual image A' is **symmetrical** to the location of the real object A with respect to the plane of the mirror. This means that:

- The line joining A and A' is perpendicular to the surface of the mirror.
- The distance from the object A to the mirror is equal to the distance from the image A' to the mirror.

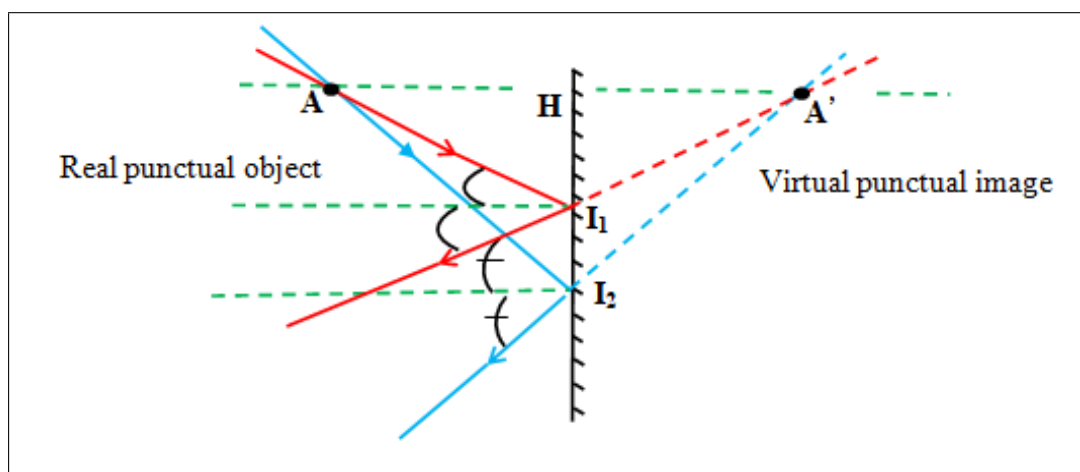


Figure 2.31: Image of a point object with plane mirror.

Conjugate relation of the plane mirror

The conjugate relation for a plane mirror is written as;

$$\overrightarrow{HA'} + \overrightarrow{HA} = 0, \quad (2.48)$$

Note

- * We use algebraic quantities (symbol by the line).

- $\overline{HA} < 0$; Object is **real**,
- $\overline{HA'} > 0$; Image is **virtual**.

The magnification (γ)

Magnification is the ratio of the size of the image to the size of object.

The magnification for a plane mirror is always equal to 1.

$$\gamma = \frac{\overline{AB}}{\overline{A'B'}} = 1. \quad (2.49)$$

Note

For a plane mirror:

- The object and its image are always of **different** nature.
- The plane mirror **preserves the size** of the objects by symmetry.

Field of vision of a plane mirror

The field of view (or field of vision) of a plane mirror is the region of space from which an observer can see a reflection in the mirror. This region is determined by the size and position of the mirror relative to the observer's eye.

To determine the field of vision, we follow these steps (Figure 2.32):

1/ **Construct the Image of the Eye:** Draw the virtual image of the observer's eye. This image will be located behind the mirror at the same perpendicular distance as the actual eye is in front of the mirror.

2/ **Trace the Limiting Rays:** From the virtual image of the eye, draw straight lines (representing the extreme rays of light that can reach the eye after reflection) that extend to the edges (contours) of the mirror.

3/ **Define the Field of Vision:** Extend these lines from the edges of the mirror outwards into the space in front of the mirror. The area enclosed by these extended lines represents the field of vision. Any object located within this region will have light rays that reflect off the mirror and appear to originate from the image of that object behind the mirror, thus being visible to the observer.

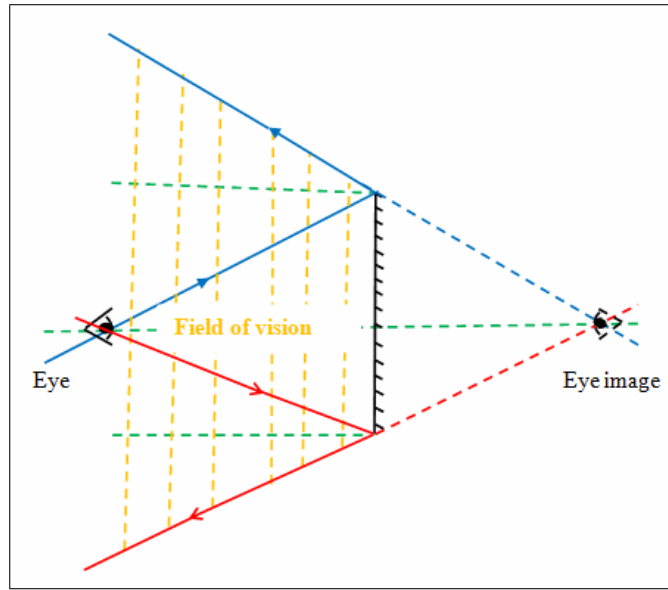


Figure 2.32: Field of vision of a plane mirror.

Example: Rearview mirror**2.6.2 Spherical mirror**

A spherical mirror is a portion of a reflecting sphere of radius R .

The radius of curvature is defined as the distance from the vertex of the mirror (S) to the center of the sphere (C):

$$\bar{R} = \bar{SC}. \quad (2.50)$$

Based on the direction of the reflecting surface, we distinguish between two types of spherical mirrors:

* **Concave Mirror:** The reflecting surface is the inner surface of the spherical section. In this convention, the center of curvature (C) is located to the left of the vertex (S), resulting in a **negative** radius of curvature (Figure 2.33):

$$\bar{R} = \bar{SC} < 0, \quad (2.51)$$

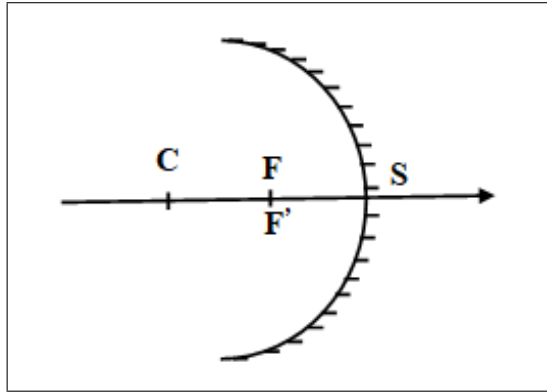


Figure 2.33: Concave Mirror.

**** Convex Mirror:** The reflecting surface is the outer surface of the spherical section. In this convention, the center of curvature (C) is located to the right of the vertex (S), resulting in a **positive** radius of curvature (Figure 2.34):

$$\bar{R} = \bar{SC} > 0, \quad (2.52)$$

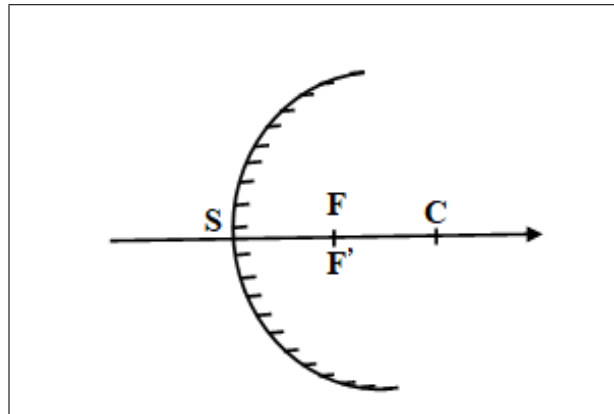


Figure 2.34: Convex Mirror.

with

C : Center of curvature of the spherical mirror.

S : Vertex of the spherical mirror.

F : Object focus.

F' : Image focus.

Convention of signs

To consistently apply the mirror equation and perform calculations related to spherical mirrors, the following sign conventions are crucial:

Direction of incident light: Light rays are assumed to travel **from left to right** towards the vertex (S).

Distances:

Distances measured to the **right** of the vertex (S) are considered **positive**.

Distances measured to the **left** of the vertex (S) are considered **negative**.

Object focus, image focus and focal length

Focal Point (F): For a spherical mirror, the object focus and the image focus coincide at a single point called the focal point (F). This is the point on the principal axis where parallel rays incident on the mirror converge (for a concave mirror) or appear to diverge from (for a convex mirror) after reflection.

Focal Length (f): The focal length (f) is the distance between the vertex of the spherical mirror (S) and the focal point (F).

Mathematically:

$$f = \bar{SF} = \bar{SF}' = \frac{\bar{SC}}{2}, \quad (2.53)$$

Note

Sign convention for focal length (f):

* **Concave Mirror:** The focal length (f) is considered negative ($f < 0$).

** **Convex Mirror:** The focal length (f) is considered positive ($f > 0$).

Image of a point object

To determine the location of the image (B') of a point object (B) formed by a spherical mirror, we trace at least two (and ideally three for verification) principal rays originating from the object point. The intersection of the reflected rays (or their apparent intersection) will locate the image point. Here are the standard principal rays used (Figures 2.31, Figure 2.32):

1/ An incident ray (R_1) passing through the center of curvature (C) of the spherical mirror strikes the mirror surface perpendicularly and is reflected back on itself (R'_1).

2/ An incident ray (R_2) traveling parallel to the principal optical axis strikes the mirror surface and is reflected (R'_2) such that it passes through the focal point (F) if the mirror is

concave, or appears to diverge from the focal point (F) located behind the mirror if the mirror is convex.

3/ An incident ray (R_3) passing through the focal point (F) (for a concave mirror) or directed towards the focal point (F) behind the mirror (for a convex mirror) strikes the mirror surface and is reflected (R'_3) travels parallel to the principal optical axis.

Graphical construction of the image of an object perpendicular to the optical axis

To graphically determine the image ($A'B'$) of an extended object (AB) that is perpendicular to the principal optical axis, where point A lies on the principal axis, we follow these steps (Figures 2.31, Figure 2.32):

1/ Use the principal ray tracing method (as described previously) to find the image (B') of the topmost point (B) of the object.

2/ The position of (A') (image of the base point (A)) is found by drawing a perpendicular line (projection) from the image point (B') to the principal axis. The point where this line intersects the principal axis is the location of (A').

3/ The image of the extended object (AB) is then the line segment connecting (A') and (B').

Examples:

1/ Concave mirror (See Figure 2,35);

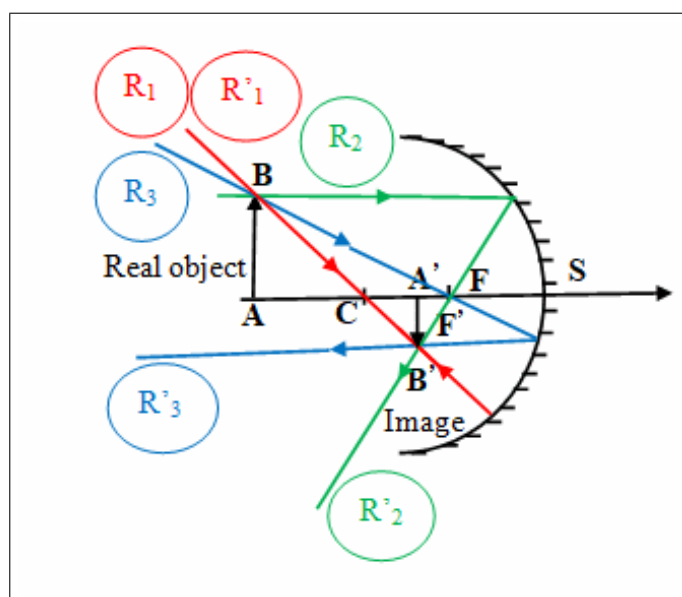


Figure 2.35: Graphical construction of the image of an object perpendicular to the optical axis (Concave mirror)

2/ Convex mirror (See Figure 2,36);

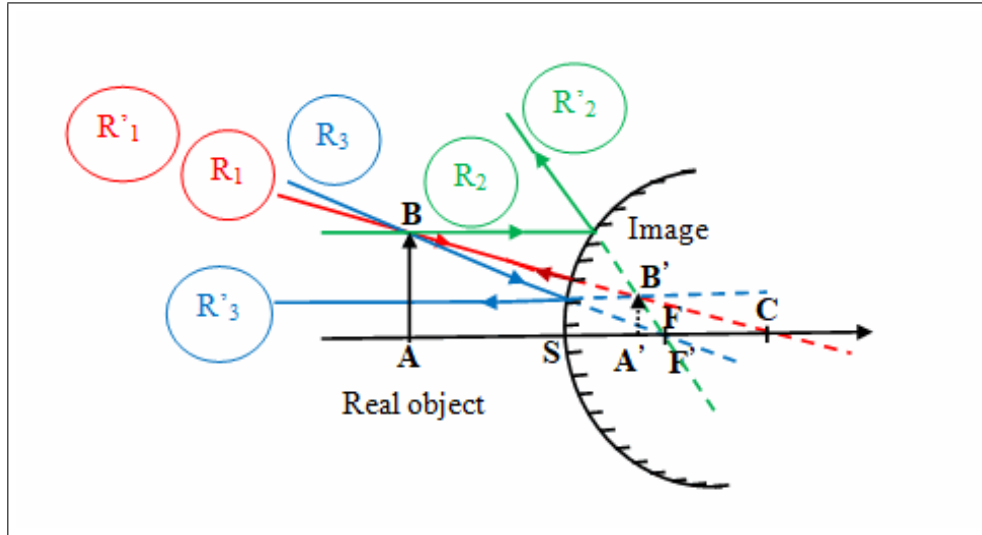


Figure 2.36: Graphical construction of the image of an object perpendicular to the optical axis (Convex mirror).

Conjugate relation

The conjugate relation (also known as the mirror equation) with respect to the vertex S is written as

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{1}{\overline{SC}}, \quad (2.54)$$

with

$\overline{SA}$: Object distance (distance of the object from the vertex S).

$\overline{SA'}$: Image distance (distance of the image from the vertex S).

$\overline{SC}$: Radius of curvature (R) of the mirror (distance of the center of curvature C from the vertex S).

It's more common to express this in terms of the focal length (f), where $f = SC/2$. Substituting this, the mirror equation becomes:

$$\frac{1}{\overline{SA'}} + \frac{1}{\overline{SA}} = \frac{1}{f}, \quad (2.55)$$

Vergence (V)

The vergence (V), also known as the optical power, of a spherical mirror is expressed as:

$$V = \frac{1}{\bar{SF}} = \frac{2}{\bar{SC}}, \quad (2.56)$$

Note**Sign Convention for Vergence:**

* **Concave mirror:** The focal length (f) is typically considered **negative**. Therefore, the vergence (V) will be **negative** ($V < 0$).

** **Convex mirror:** The focal length (f) is typically considered **positive**. Therefore, the vergence (V) will be **positive** ($V > 0$).

The vergence is expressed in **diopeters** (δ), where $1 \text{ diopter} = 1m^{-1}$.

Magnification

The magnification (γ) of a spherical mirror given by;

$$\gamma = \frac{\bar{A'B'}}{\bar{AB}} = -\frac{\bar{SA'}}{\bar{SA}}, \quad (2.57)$$

Nature of the image

The nature of the image can be determined based on the **sign** of the magnification (γ) and the **sign** of the image distance $\left(\bar{SA'}\right)$, following the same sign conventions used in the mirror equation:

*** Magnification (γ):**

- $\gamma > 0$: **Upright** image (relative to the object).
- $\gamma < 0$: **Inverted** image (relative to the object).
- $|\gamma| > 1$: **Enlarged** image (larger than the object).
- $|\gamma| < 1$: **Reduced** image (smaller than the object).
- $|\gamma| = 1$: Image is the **same size** as the object.

**** Image distance $\left(\bar{SA'}\right)$:**

$\bar{SA'} > 0$: **Real** image .

$\bar{SA'} < 0$: **Virtual** image.

2.7 Optical Instruments

2.7.1 The Eye

The model of the eye (Reduced eye)

The reduced eye model simplifies the eye's complex structure by representing it **as a single converging lens and a screen**, analogous to a camera. The **iris** acts as a **diaphragm**, controlling the amount of light entering the eye, while the **cornea** and **lens** together function as a **converging lens**, focusing light onto the **retina**, which acts as the **screen** (Figure 2.37).

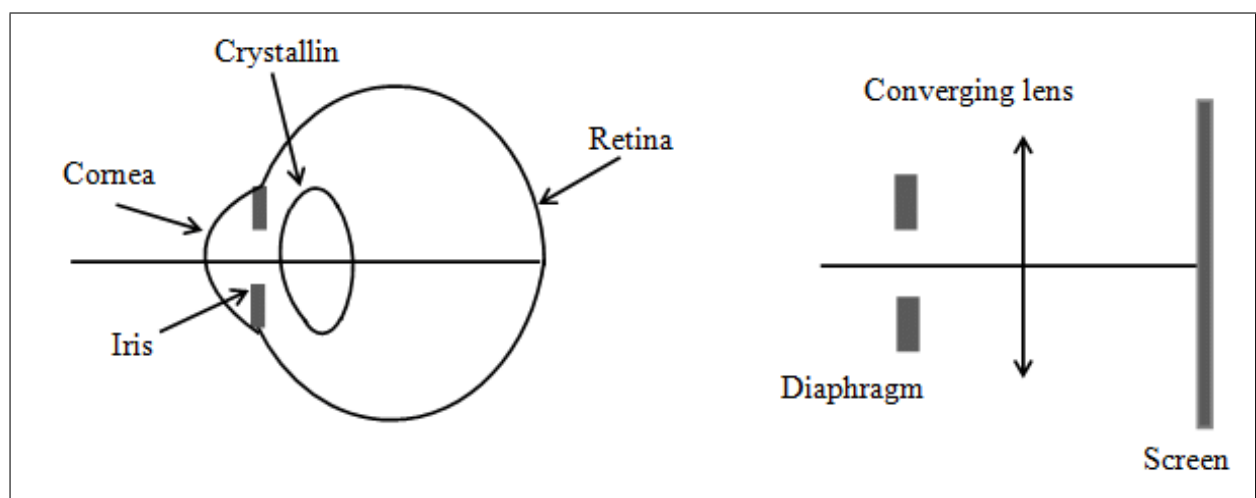


Figure 2.37: Reduced eye.

See an object clearly

To always see an object clearly regardless of its distance, the lens can change (Change of curvature), deform, and alter its focal length; this is accommodation. The image formed on the retina is inverted (See Figure 2.38).

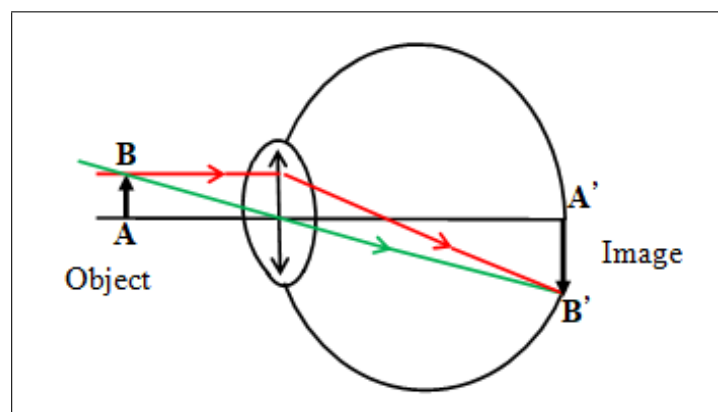


Figure 2.38: Emmetropic eye (see an object clearly).

The normal eye (Emmetropic eye)

An emmetropic eye (normal eye) is defined as an eye without refractive error (See Figure 2.38).

Punctum Proximum (PP), Punctum Remotum (PR) and field of vision

* **Punctum Proximum (PP)**: is the **closest point** (or **near point**) at which the eye can focus clearly with **maximum accommodation**.

* **Punctum Remotum (PR)**: is the most **distant point** (or **Far Point**) that an eye sees clearly **without accommodating (at rest)**.

* **Field of vision**: The range of clear vision (or the accommodative range) is limited by the Punctum Proximum (PP) and the Punctum Remotum (PR) (See Figure 2.39).

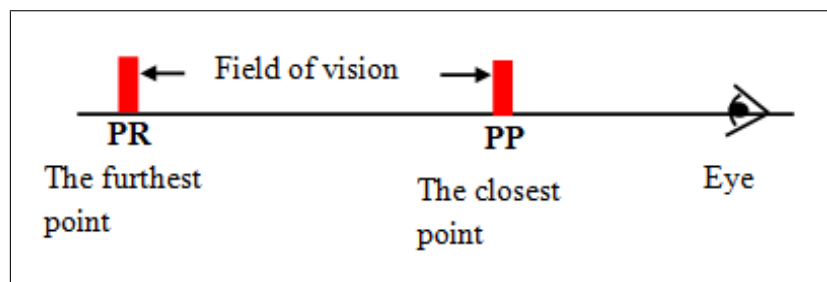


Figure 2.39: Field of vision.

Note:

* For a normal eye, emmetropic eye, the Punctum Remotum (PR) is considered to be at infinity (∞), and the Punctum Proximum (PP) is typically around 25 centimeters (cm)(See Figure 2.40).

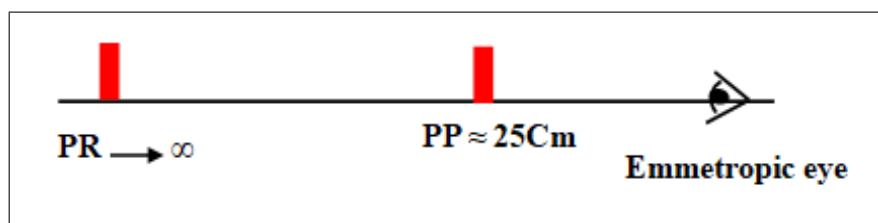


Figure 2.40: Field of vision (Emmetropic eye).

Eye defects and correction

Presbyopia: Presbyopia is related to the progressive loss of accommodation due to age, specifically a decrease in the lens's elasticity. This age-related loss of lens elasticity causes the Punctum Proximum (PP) to recede (See Figure 2.41).

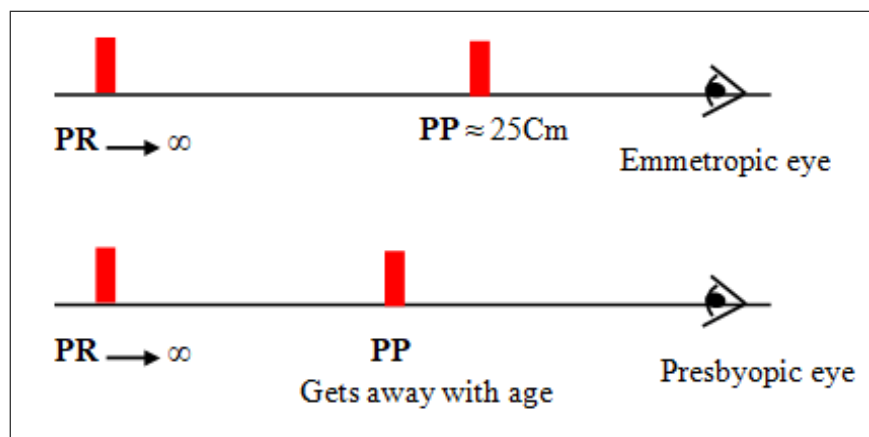


Figure 2.41: Field of vision (Presbyopia).

Correction

To correct this defect (**presbyopia**), a **converging lens** is added. In this case, the **Punctum Proximum (PP)** is corrected.

Myopia:

A myopic eye is **excessively convergent**, causing the image of a distant object to form in **front** of the retina (See Figure 2.42). Myopia results in both the Punctum Proximum (PP) and the Punctum Remotum (PR) moving **closer** (See Figure 2.43).

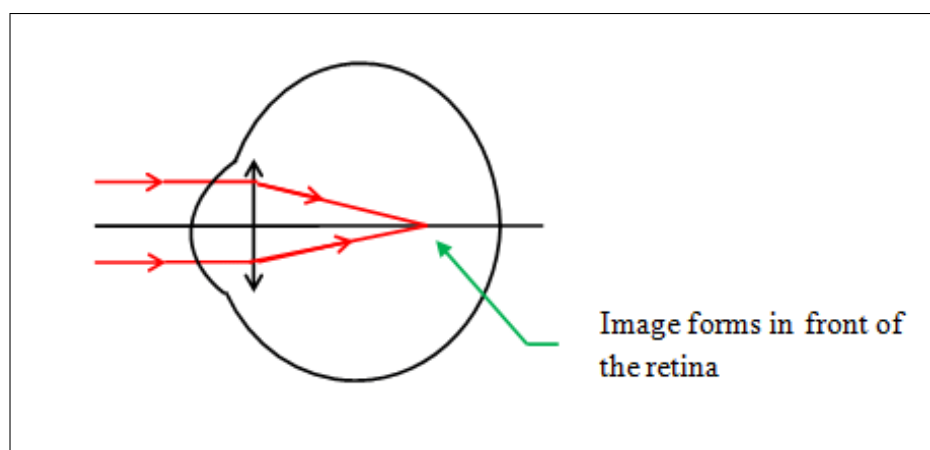


Figure 2.42: Myopia.

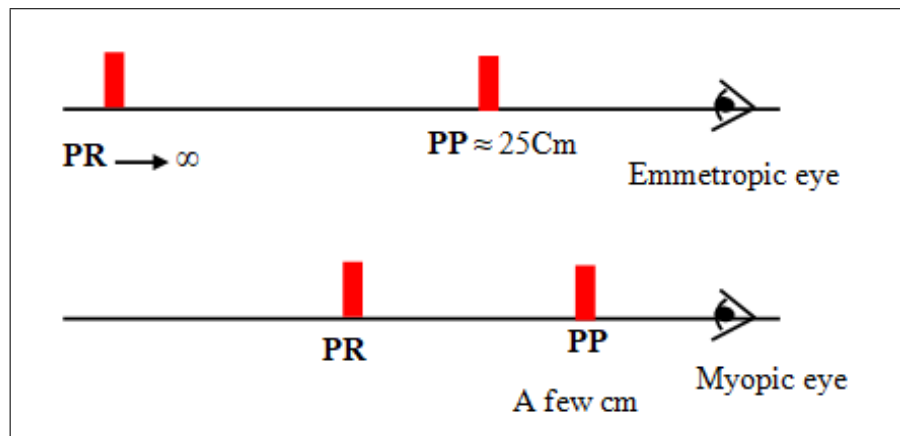


Figure 2.43: Field of vision (Miopia).

Correction:

A **diverging lens** must be added to allow the image to form on the retina (See Figure 2.44). We correct the PP and PR.

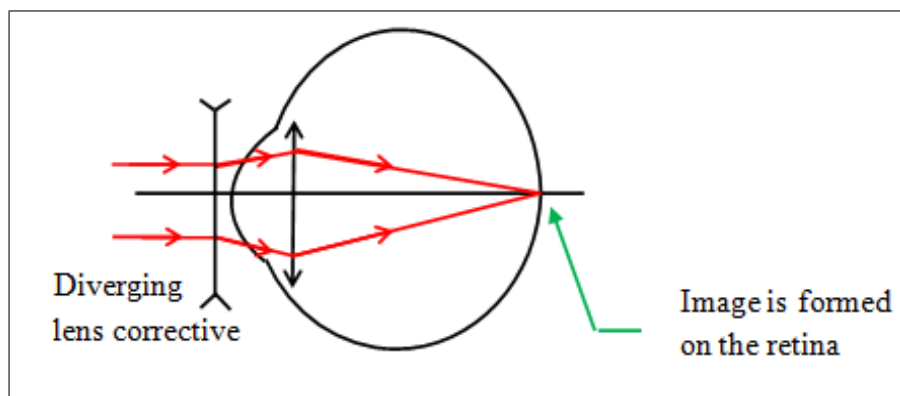


Figure 2.44: Myopia correction.

Hyperopia:

A hyperopic eye is **insufficiently convergent**, causing the image of a distant object to form **behind** the retina (See Figure 2.45). Hyperopia causes the Punctum Proximum (*PP*) to **recede** (See Figure 2.46).

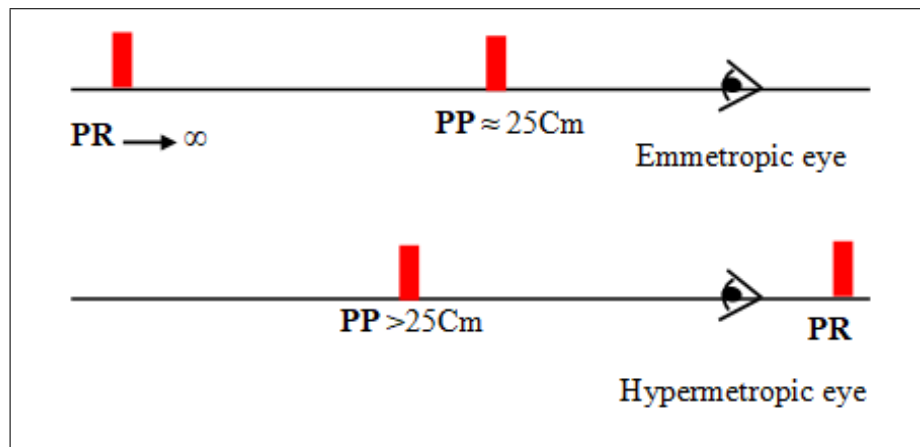


Figure 2.1: Figure 2.46: Field of vision (Hypermetropia).

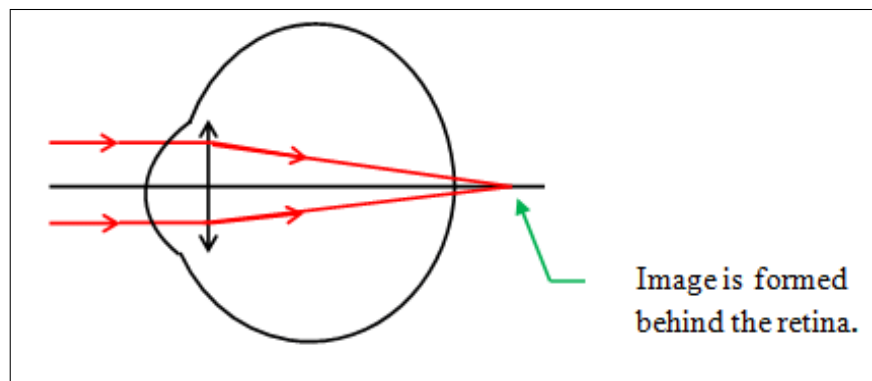


Figure 2.45: Hyperopia.

Correction

A **converging** lens must be added to allow the image to form on the retina (Figure 2.47). We correct the *PR*.

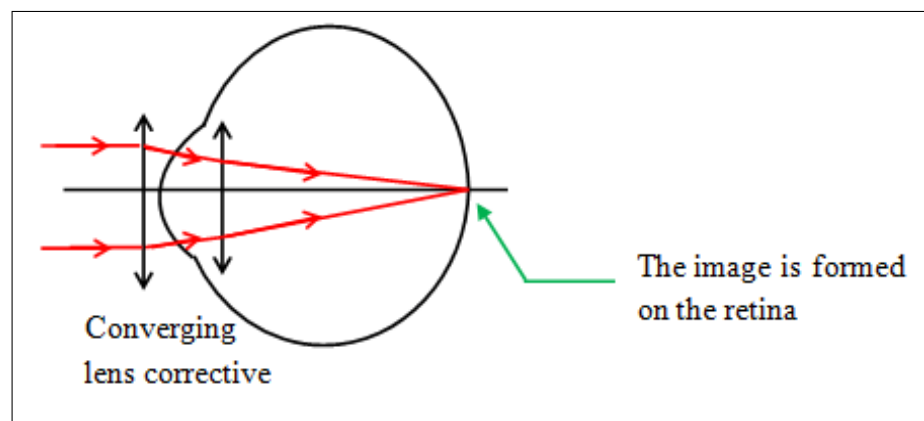


Figure 2.47: Hyperopia correction.

Note

Accommodation is required to see virtual objects located behind the Punctum Remotum (PR) and real points located in front of the Punctum Proximum (PP).

Amplitude:

The dioptric amplitude (A) measures the vergence (V) required to form an image of the Punctum Proximum (PP) at the Punctum Remotum (PR);

$$A = \frac{1}{\sigma} - \frac{1}{\Delta}, \quad (2.58)$$

where

$$\sigma = |PP|, \quad \Delta = |PR|. \quad (2.59)$$

* The dioptric amplitude remains constant for an eye to which a lens of vergence (V) is added;

$$\text{Amplitude before correction} = \text{Amplitude after correction.}$$

The power of the corrective lens:

The power of the corrective lens is written as:

$$V = \frac{1}{\overline{OA_i}} - \frac{1}{\overline{OA_f}}, \quad (2.60)$$

with

V : The power of the corrective lens is measured in diopters (δ).

* When correcting the Punctum Proximum (PP), we take:

$\overline{OA_i}$: Position of (PP) before correction.

$\overline{OA_f}$: Position of (PP) after correction.

** When correcting the Punctum Remotum (PR), we take:

$\overline{OA_i}$: Position of (PR) before correction.

$\overline{OA_f}$: Position of (PR) after correction.

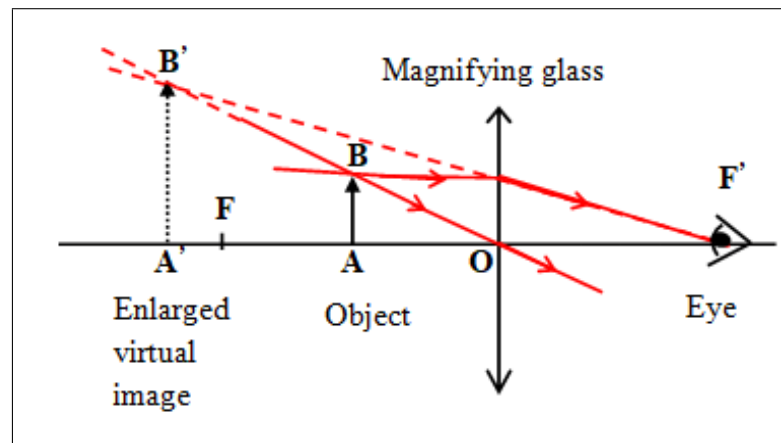


Figure 2.2: Figure 2.48: Magnifying glass.

Focal length

The focal length of the corrective lens written as;

$$f' = \frac{1}{V}, \quad (2.61)$$

where

f' is the focal length and V is the **power** of the corrective lens.

Note

The **sign** of the focal length (f') of the corrective lens determines its type:

- $f' > 0$, Converging lens.
- $f' < 0$, Diverging lens.

2.7.2 The magnifying glass and the optical microscope

Magnifying glass

A magnifying glass is a thick converging lens with a short focal length (f'), typically between 2cm and 10cm . It is used by **myopic** or **emmetropic** eyes and produces an **enlarged virtual image** of the object (See Figure 2.48).

Object position The object to be examined must be placed between the magnifying glass and its object focal point (F). In other words, the distance between the object and the lens must be less than the focal length of the lens.

Image formation When the object is placed between the magnifying glass and its object focal point, the **magnifying glass** forms a **virtual, upright, and magnified image**.

Focusing * Focusing involves adjusting the distance between the object and the magnifying glass so that the virtual image formed by the magnifying glass is clear to the observer's eye.

* To obtain a clear image, the virtual image must be within the eye's range of distinct vision, i.e., between the Punctum Proximum (PP) and the Punctum Remotum (PR).

Distance adjustment * By changing the distance between the object and the magnifying glass, the position of the virtual image is changed.

* The points A_i and A_f represent the extreme positions of the object which correspond respectively to the Punctums Remotum (PR) and Proximum (PP) of the eye.

* A_i has PR as image, A_f has PP as image.

Eye accommodation latitude * The eye accommodation latitude L , in the context of using a magnifying glass, is the distance between the extreme positions (A_i and A_f) of the object for which the image formed by the magnifying glass remains clear to the observer.

* In other words, it is the interval within which the object can be moved while maintaining a clear image (it must be located between the Punctum Proximum (PP) and the Punctum Remotum (PR) of the observer's eye) (See Figure 2.49).

The eye accommodation latitude L is written as

$$L = \left| \bar{OA}_f - \bar{OA}_i \right|, \quad (2.62)$$

where

OA_i : Object position for image at Punctum Remotum (PR).

$\bar{OA}_f$: Object position for image at Punctum Proximum (PP).

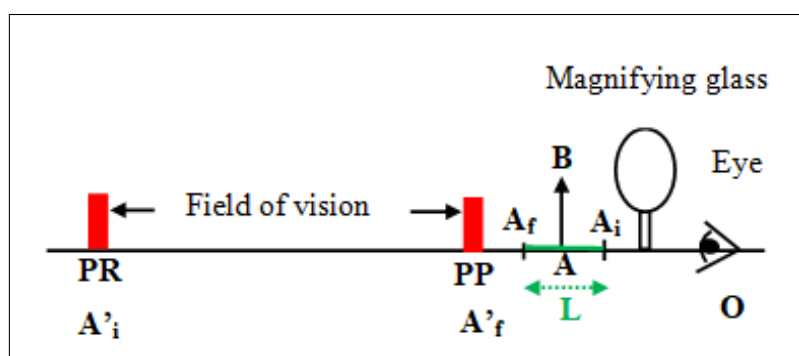


Figure 2.49: Eye accommodation latitude.

Power of a magnifying glass The power P of a magnifying glass is defined as follows;

$$P = \frac{\beta}{\bar{AB}}, \quad (2.63)$$

with

P : Power in diopters (δ).

β : Diameter in radians (rad).

$\bar{AB}$: Object size in meters (m).

Note

When the object is placed in the object focal plane F , its image is formed at **infinity**. In this case, its diameter β is expressed as follows;

$$\beta = \frac{\bar{AB}}{OF'} = \frac{\bar{AB}}{f'}, \quad (2.64)$$

where

$$P = \frac{\beta}{\bar{AB}} = \frac{1}{f'} = V, \quad (2.65)$$

with V is the vergence, f' is the focal distance.

Magnification :

Magnification G is defined as follows;

$$G = \frac{\beta}{\alpha} = \frac{\beta}{\bar{AB}} \times \frac{\bar{AB}}{\alpha} = P \times d_m, \quad (2.66)$$

with

G : Grossissement.

P : Power in diopters (δ).

d_m : Minimum distance between the object and the eye.

β : Apparent diameter of the image $A'B'$ in radians (rad).

α : Apparent diameter of the object AB in radians (rad) .

The microscope:

A microscope is a **very high magnification** instrument comprising an **objective**, similar to a very **convergent thin lens**, and an **eyepiece** acting as a **magnifying glass** in the examination of the real, very **enlarged image** that the objective gives of the object examined (See Figure 2.50).

The microscope is the combination of two convergent systems. It is used to observe small, close objects.

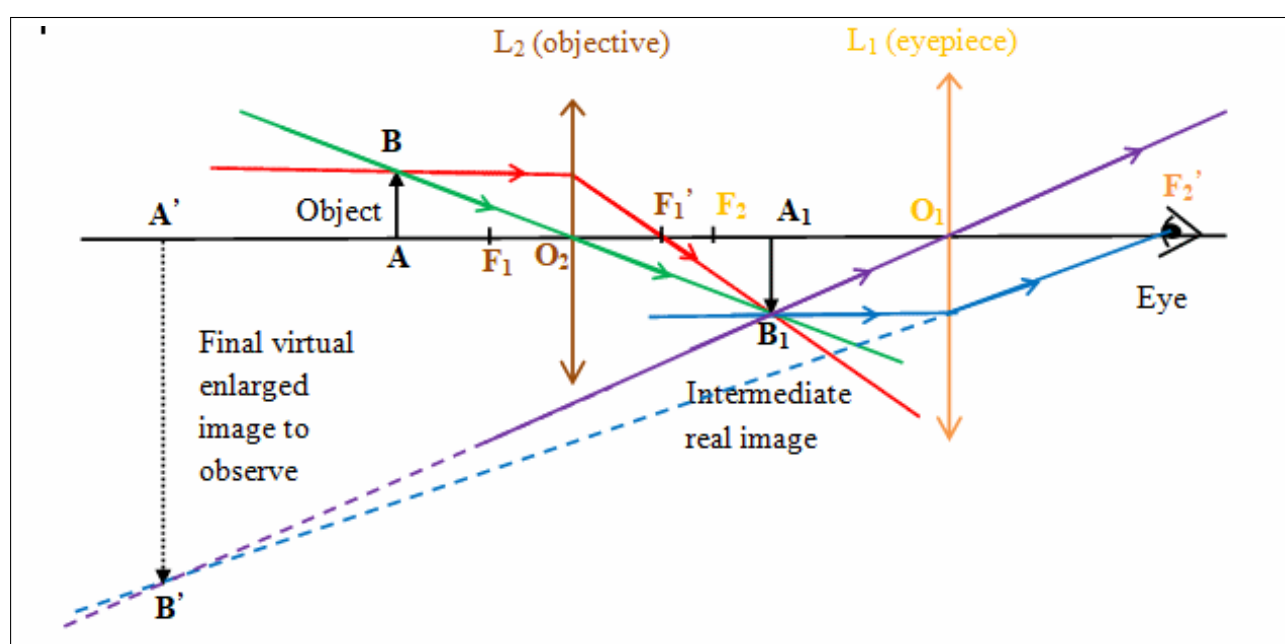


Figure 2.50: The microscope.

Note

The focal length of a microscope **objective** is significantly **shorter** than that of its **eyepiece**.

Power The power of a microscope given by;

$$P = \frac{\beta}{AB}, \quad (2.67)$$

with

P : Power in diopters (δ).

β : The apparent diameter of the image in radians (rd).

$\bar{AB}$: Object size in meters (m).

The power of the eyepiece reads;

$$P_m = \frac{\bar{A_1B_1}}{\bar{AB}} \times \frac{\beta}{\bar{A_1B_1}} = \gamma_1 \times P_o, \quad (2.68)$$

with

P_m : Power of the eyepiece.

P_o : Power of the eyepiece.

γ_1 : Magnification of the objective.

β : Apparent diameter of the image in radians (rd).

$\bar{AB}$: Object size in meters (m).

$\bar{A_1B_1}$: Intermediate image size in meters (m).

Magnification The magnification of the microscope is written as

$$G = \frac{\beta}{\alpha} = \frac{\beta}{\bar{AB}} \times \frac{\bar{AB}}{\alpha} = P_m \times d_m, \quad (2.69)$$

where d_m is the minimum distance of distinct vision.

The focusing latitude of the microscope It has been previously explained regarding magnification, therefore it is given with

$$L = \left| \bar{OA_f} - \bar{OA_i} \right|, \quad (2.70)$$

with

L : The focusing latitude of the microscope.

$\bar{OA_i}$: The position of the object relative to the objective when the position of its image is formed on the near point (PP) of the eye.

$\bar{OA_f}$: The position of the object relative to the objective when the position of its image is formed on the far point (PR) of the eye.

2.8 Exercises

Exercise 1:

Calculate the velocity of light in glass with a refractive index $3/2$.

Exercise 2:

A ray of light passes from air into water with a refractive index $n = 1.33$. The angle of incidence is 45° .

- 1/ Calculate the angle of refraction (r).
- 2/ Deduce the angle of deviation (D).
- 3/ Draw the rays satisfying the laws of refraction.

Exercise 3:

A ray of light passes from glass with a refractive index $n = 1.5$ into air. The angle of incidence is 60° .

- 1/ What is the phenomena occur (refraction or reflection).
- 2/ Draw the rays satisfying the laws of refraction or reflection.

Exercise 4:

Complete the path of the light ray through an isosceles right-angled prism with refractive index $n = 1.5$ immersed in air (Figure 2.51).

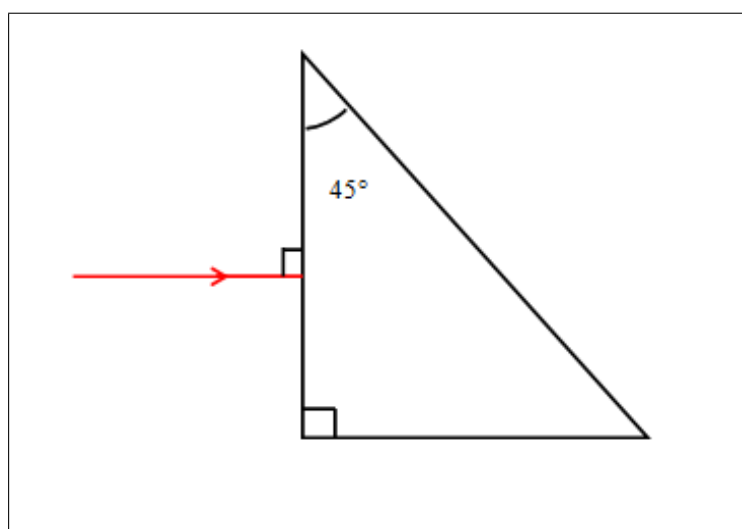


Figure 2.51: Isosel rectangle prism.

Exercise 5:

An object, 1cm in size, is placed at 5cm in front of a concave mirror of focal length 3cm .

- 1/ Find the position, nature and the size of the image.
- 2/ Draw the ray diagram.

Exercise 6:

A concave spherical diopter with a radius of curvature of 4cm separates two media with refractive indices $n_1 = 1$ and $n_2 = 2$.

- 1/ Calculate the positions of the object and image focal points and determine the nature of the diopter.
- 2/ Trace the image formed by the diopter of a straight object AB , 1cm in height, located 12cm in front of the vertex S .
- 3/ Perform the geometric construction.

Exercise 7:

A 2cm tall object is placed perpendicular to the principal axis of a converging lens of focal length 5cm . The distance of the object from the lens is 10cm .

- 1/ Find the nature, position and size of the image. Also find its magnification.
- 2/ Draw the ray diagram

Exercise 8:

Consider a real and upright object with a height of 1cm placed 6cm behind a diverging lens with a focal length of 2cm .

- 1/ Deduce the vergence (optical power) of this lens.
- 2/ Find the size, position, and nature (real or virtual, upright or inverted) of the image.
- 3/ Verify the results using a geometric construction (ray diagram).

Exercise 9:

A presbyopic eye has amplitude of accommodation equal to 2.5Diopter and its far point (Punctum Remotum) at infinity.

- 1/ Calculate the position of its near point (Punctum Proximum).
- 2/ What is the focal length of the corrective lens that should be associated with this eye so that it can read at 22cm with maximum accommodation? Deduce the nature of this lens.

Exercise 10:

A hyperopic eye is exactly corrected by a lens with a vergence of 4D (*Diopters*). Deduce the position of its near point (Punctum Proximum).

Exercise 11:

A myopic eye can only see distant objects clearly up to a distance of $4m$. Its near point (Punctum Proximum) is at $18cm$.

- 1/ Calculate the focal length of the corrective lens required to see distant objects clearly.
- 2/ Deduce the nature of this lens.
- 3/ What is the range of clear vision (field of clear vision) before and after correction?

Exercise 12:

A magnifying glass is a lens with a focal length $f = 4cm$. Consider a normal eye of an observer placed at the image focal point of the magnifying glass, whose minimum distance of distinct vision is $14cm$.

What is the focusing range (latitude of focusing) of the magnifying glass with respect to the observer's eye?

Exercise 13:

A microscope contains two lenses: an eyepiece with a focal length of $f_1 = 4mm$ and an objective with a focal length of $f_2 = 12mm$. The distance between the two lenses is $16cm$.

Consider a normal eye of an observer placed at the image focal point of the eyepiece, whose minimum distance of distinct vision is $14cm$.

- 1/ Where should the object be placed so that the final image is formed at infinity?
- 2/ What should be the new position of the object so that the final image is formed at the minimum distance of distinct vision?
- 3 / Deduce the focusing range (latitude of focusing) of the microscope.

2.9 Solutions

Exercise 1:

The velocity of light in glass writes (2.2);

$$v = \frac{c}{n}, \quad (2.71)$$

$$v = \frac{3 \times 10^8 (m/s)}{1.5} = 2 \times 10^8 (m/s). \quad (2.72)$$

Exercise 2:

- 1/Angle of refraction (r):

The light passes from air into water, then it passes from **rarer** medium do **denser** medium. In this case we have the refraction phenomena. To find the angle of refraction appalling the Snell'Discart's low of refraction (2.4) ,then

$$r = \text{Arc sin} \left[\frac{1}{1.33} \sin (45^\circ) \right], \quad (2.73)$$

$$= 32.13^\circ. \quad (2.74)$$

2/ Angle of deviation (D):

$$D = i - r, \quad (2.75)$$

$$= 45^\circ - 32.13^\circ, \quad (2.76)$$

$$= 12.87^\circ. \quad (2.77)$$

3/ Ray diagram (Figure 2.52):

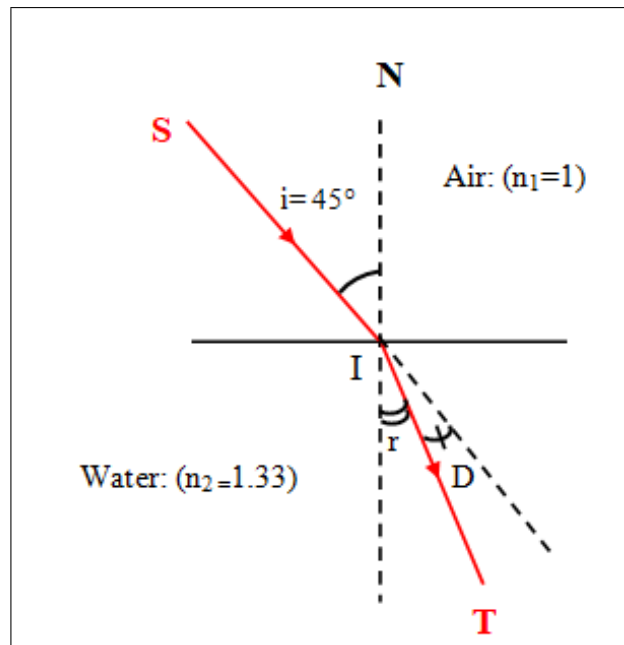


Figure 2.52: Refraction; Air-Water.

Exercise 3:

1/ The phenomena occur:

The light passes from **dense** medium (Glass) to **rarer** medium (Air). It is necessary to calculate the crucial angle (i_l) (2.5);

$$i_l = \text{Arc sin} \left(\frac{n_{air}}{n_{glass}} \right), \quad (2.78)$$

$$= \text{Arc sin} \left(\frac{1}{1.5} \right), \quad (2.79)$$

$$= 41.8^\circ. \quad (2.80)$$

Her the angle of incidence (i) is greater then the critical angle (i_l); ($i > i_l$) the phenomena of **reflection** occur.

Applying the Snell'Discart's low of reflection (2.6), then the angle of reflection is;

$$i' = i = 60^\circ. \quad (2.81)$$

2/ Ray diagram (Figure 2.53);

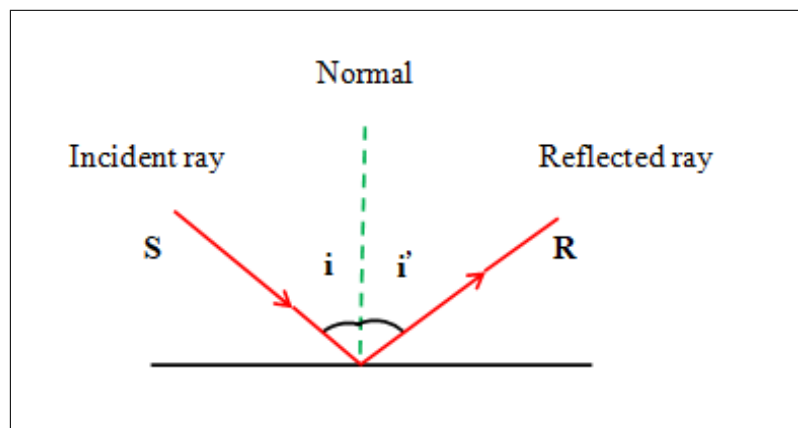


Figure 2.53: Reflection Glass-Air.

Exercise 4:

1/ To complete the path of the light ray through the prism we apply the Snell's low of refraction:

Face 1: Air-Prisme: The ray don't deviated (Normal incidence).

$$i = r = 0, \quad (2.82)$$

Face 2: Prisme-Air:

The angle of refraction (r') :

$$r' = 45^\circ, \quad (2.83)$$

The critical angle (i_l);

$$i_l = \text{Arcsin} \left(\frac{n_{air}}{n_{prism}} \right), \quad (2.84)$$

$$= \text{Arcsin} \left(\frac{1}{1.5} \right), \quad (2.85)$$

$$= 41.8^\circ. \quad (2.86)$$

In this face we have total internal **reflection** ($r' > i_l$), then the angle of reflection (r'') reads:

$$r'' = r' = 45^\circ, \quad (2.87)$$

Face 3: Prisme-Air

The ray don't deviated (Normal incidence).

$$i' = r''' = 0, \quad (2.88)$$

2/ Ray diagram (Figure 2.54);

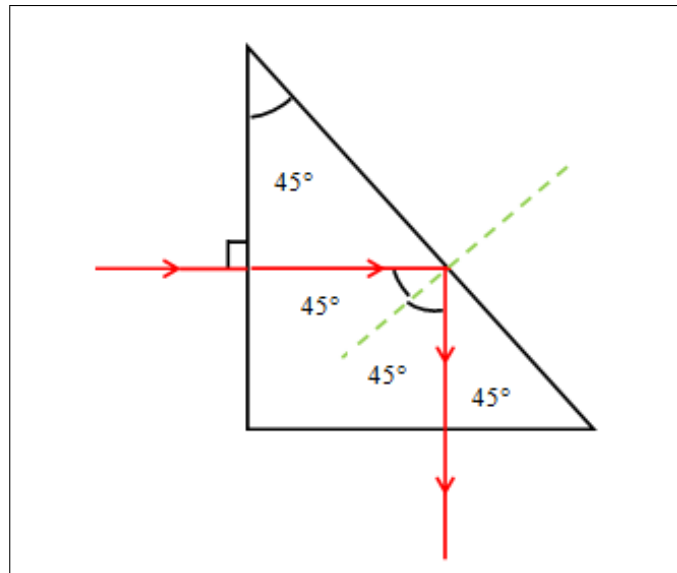


Figure 2.54: Prisme rectangle isosel (Ray diagramme)

Exercise 5:

$$\bar{SA} = -5cm, \bar{SA} = -6cm$$

$$SC = 2\bar{SF} = 2\bar{SF}' = -2 \times 3cm = -6cm, \quad (2.89)$$

$$\frac{1}{\bar{SA}'} = -\frac{1}{\bar{SA}} + \frac{2}{\bar{SC}}, \quad (2.90)$$

$$\frac{1}{\bar{SA}'} = \frac{-1}{-5cm} + \frac{2}{-6cm}, \quad (2.91)$$

$$\bar{SA}' = -7.5cm. \quad (2.92)$$

$\bar{SA}' < 0$, Image is **real**.

Magnification (γ)

$$\gamma = \frac{\bar{A'B'}}{\bar{AB}} = -\frac{\bar{SA}'}{\bar{SA}}, \quad (2.93)$$

$$= \frac{-7.5cm}{-5cm} = +1.5. \quad (2.94)$$

- $\gamma > 0$: **Upright** image (relative to the object).
- $|\gamma| > 1$: **Enlarged** image (larger than the object).

Nature of image: Image is **real, upright and enlarged**.

* Size of the image;

$$\bar{A'B'} = \gamma \bar{AB} \quad (2.95)$$

$$= (+1.5)(1cm) = +1.5cm. \quad (2.96)$$

2/ Geometric construction (Figure 2.55);

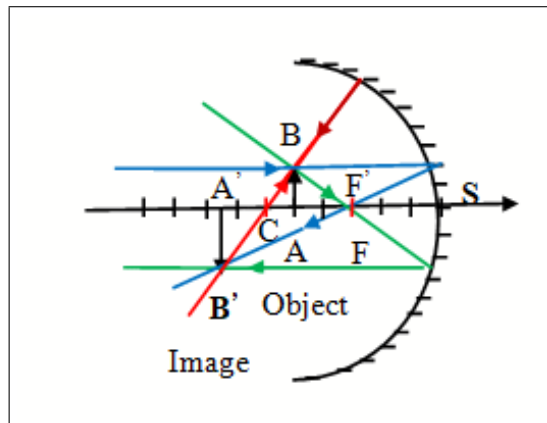


Figure 2.55: Geometric construction (Spherical mirror).

Exercise 6:

$R = \bar{SC} = -4cm$ (Concave refractive spherical surface).

1/ Object and image focal points

$$\bar{SF} = \frac{n_1}{n_1 - n_2} \bar{SC} = \frac{1}{1 - 2} (-4cm) = +4cm, \quad (2.97)$$

$$\bar{SF}' = \frac{n_2}{n_2 - n_1} \bar{SC} = \frac{2}{2 - 1} (-4cm) = -8cm. \quad (2.98)$$

Here $\bar{SF} > 0$ and $\bar{SF}' < 0$ the diopter is divergent.

2/ Nature of the image:

$\bar{SA} = -12cm$ (In front of the vertex S).

* Position of image

Using the conjugate relation (2.32) , the position of the image reads

$$\frac{\frac{n_2}{\bar{SA}'}}{\frac{n_1}{\bar{SA}}} = \frac{n_1}{\bar{SA}} + \frac{n_2 - n_1}{\bar{SC}}, \quad (2.99)$$

$$\frac{\frac{2}{\bar{SA}'}}{\frac{1}{-12cm}} = \frac{1}{-12cm} + \frac{2 - 1}{-4cm}, \quad (2.100)$$

$$\bar{SA}' = -6cm. \quad (2.101)$$

$\bar{SA}' < 0$, the image is **virtual**.

* The magnification (γ) is defined as:

$$\gamma = \frac{\bar{A'B'}}{\bar{AB}} = \frac{n_1}{n_2} \frac{\bar{SA'}}{\bar{SA}}, \quad (2.102)$$

$$= \frac{1}{2} \frac{(-6)}{(-12)} = 0.25. \quad (2.103)$$

$\gamma < 0$, the image is **inverted**.

$|\gamma| < 1$: The image is **reduced**.

Nature of the image; **virtual, inverted** and **reduced**.

* Image height $\left(\bar{A'B'}\right)$:

$$\bar{A'B'} = \gamma \bar{AB} = 0.25 \times 1cm = 0.25cm. \quad (2.104)$$

3/ Geometric construction (Figure 2.56);

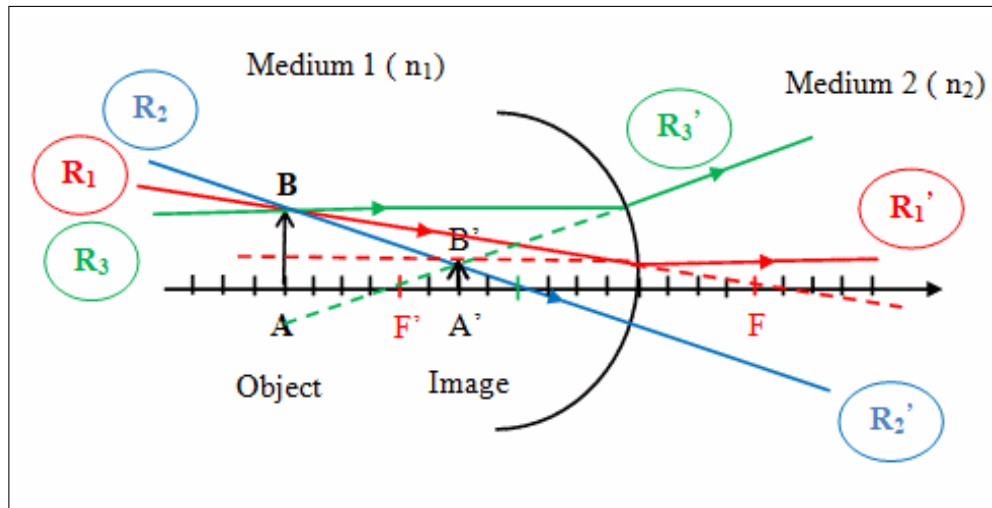


Figure 2.56: Geometric construction (Spherical refracting surface).

Exercise 7:

$$\bar{OA} = -10cm, f' = +5cm.$$

1/ The nature, position and size of the image

Using the conjugate relation (2.44), we find

$$\frac{1}{\bar{OA}'} = \frac{1}{\bar{OA}} + \frac{1}{f'}, \quad (2.105)$$

$$\frac{1}{\bar{OA}'} = \frac{1}{-10cm} + \frac{1}{+5cm}, \quad (2.106)$$

$$\bar{OA}' = +10cm. \quad (2.107)$$

$\bar{OA}' > 0$, the image is **real**.

The magnification (γ) is defined as (2.45):

$$\gamma = \frac{\bar{A'B'}}{\bar{AB}} = \frac{\bar{OA}'}{\bar{OA}}, \quad (2.108)$$

$$= \frac{(+10cm)}{(-10cm)} = -1. \quad (2.109)$$

$\gamma < 0$, the image is **inverted**.

$|\gamma| = 1$: The image is the **same size** as the object.

Nature of the image; **real, inverted** and the **same size** as the object.

2/ Geometric construction (Figure 2.57);

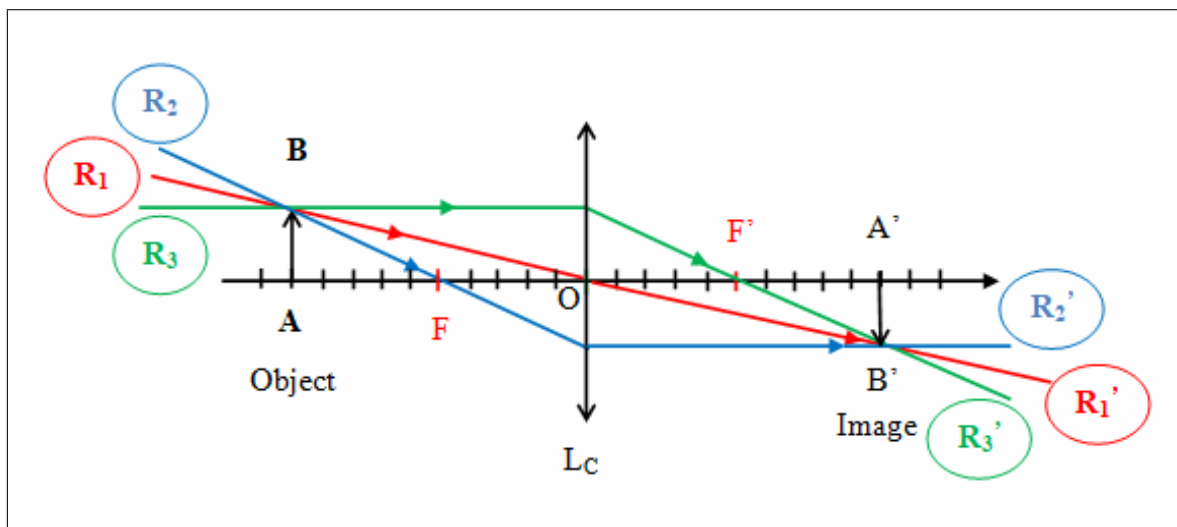


Figure 2.57: (Geometric construction) Converging lens.

Exercise 8:

1/ The nature, position and size of the image

Using the conjugate relation (2.44), we find

$$\frac{1}{\bar{OA}'} = \frac{1}{\bar{OA}} + \frac{1}{f'}, \quad (2.110)$$

$$\frac{1}{\bar{OA}'} = \frac{1}{+6cm} + \frac{1}{-2cm}, \quad (2.111)$$

$$\bar{OA}' = -3cm. \quad (2.112)$$

$\bar{OA}' > 0$, the image is **real**.

The magnification (γ) is defined as (2.45):

$$\gamma = \frac{\bar{A'B'}}{\bar{AB}} = \frac{\bar{OA}'}{\bar{OA}}, \quad (2.113)$$

$$= \frac{(-3cm)}{(6cm)} = -0.5. \quad (2.114)$$

$\gamma < 0$, the image is **inverted**.

$|\gamma| < 1$: The image is **reduced**.

Nature of the image; **real**, **inverted** and **reduced**.

2/ Geometric construction (Figure 2.58);

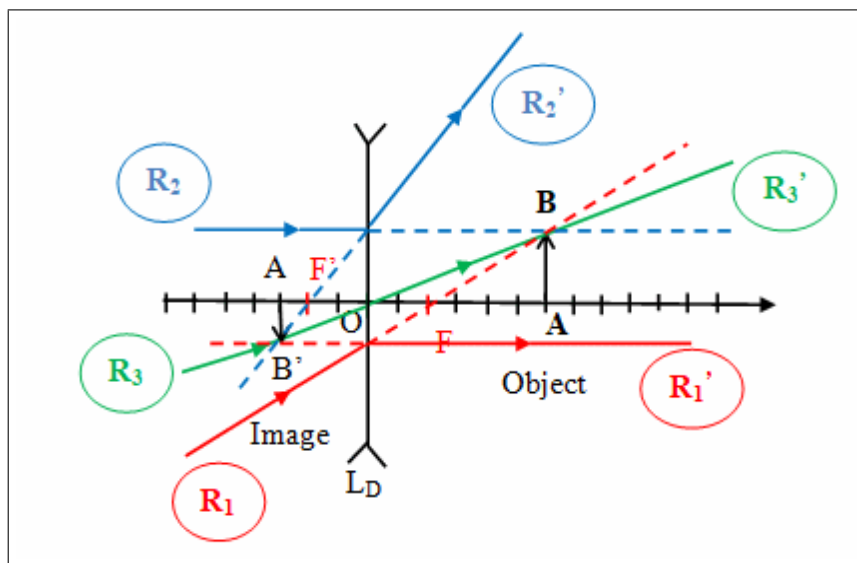


Figure 2.58: Geometric construction (Converging lens).

Exercise 9:

$$A = 2.5D, PR \rightarrow -\infty.$$

1/Punctum Proximum (PP)

Using (2.58), the Punctum Proximum reads

$$\frac{1}{\sigma} = A + \frac{1}{\Delta}, \quad (2.115)$$

$$= 2.5D + \frac{1}{\infty}, \quad (2.116)$$

$$\sigma = 0.4m = 40cm. \quad (2.117)$$

then

$$|PP| = \sigma = 40cm. \quad (2.118)$$

2/ The focal length of the corrective lens that should be associated with this eye so that it can read at $22cm$ with maximum accommodation

* For the presbyopic eye we correct its PP .

The focal length of the corrective lens is written as;

$$\frac{1}{f'} = \frac{1}{\overline{OA_i}} - \frac{1}{\overline{OA_f}}, \quad (2.119)$$

$$\frac{1}{f'} = \frac{1}{-40cm} - \frac{1}{-22cm}, \quad (2.120)$$

$$f' = -48.88cm. \quad (2.121)$$

with

$\overline{OA_i} = -40cm = 0$, position of (PP) before correction.

$\overline{OA_f} = -22cm$, position of (PP) after correction.

* Nature of the corrected lens

$f' < 0$, the corrected lens is convergent.

Exercise 10:

Position of its near point (Punctum Proximum).

$$\frac{1}{\sigma} = A + \frac{1}{\Delta}, \quad (2.122)$$

$$= 4 + \frac{1}{\infty}, \quad (2.123)$$

$$\sigma = 0.25m = 25cm. \quad (2.124)$$

$$|PP| = \sigma = 25cm. \quad (2.125)$$

Exercise 11:

1/ Focal length of the corrective lens required to see distant objects clearly

$$\frac{1}{f'} = \frac{1}{\overline{OA_i}} - \frac{1}{\overline{OA_f}}, \quad (2.126)$$

$$\frac{1}{f'} = \frac{1}{4m} - \frac{1}{\infty}, \quad (2.127)$$

$$f' = 0.25D. \quad (2.128)$$

with

$\overline{OA_i} = 4m$, position of (PR) before correction.

$\overline{OA_f} = -\infty$, position of (PR) after correction.

2/ Nature of lens;

$f' > 0$, the corrected lens is divergent.

Field of clear vision before and after correction.

* Before correction;

$$[PR_{nc}, PP_{nc}] = [-4m, -0.18m]$$

* After correction;

$$]-\infty, PP_c = ?[$$

$$A = \frac{1}{\sigma} - \frac{1}{\Delta} = \frac{1}{\sigma'} - \frac{1}{\Delta'}, \quad (2.129)$$

where

$$\sigma = |PP_{nc}| = 18cm = 0.18m, \sigma' = |PP_c| = ?, \Delta = |PR_{nc}| = 4m \text{ and } \Delta' = |PR_c| = \infty.$$

Then

$$\frac{1}{\sigma'} = \frac{1}{\sigma} - \frac{1}{\Delta} + \frac{1}{\Delta'}, \quad (2.130)$$

$$= \frac{1}{18cm} - \frac{1}{400cm} + \frac{1}{\infty}, \quad (2.131)$$

$$\sigma' \simeq 18.85cm. \quad (2.132)$$

* Field of clear vision before and after correction is $]PR_c, PP_c] =]-\infty, -18.85cm]$.

Exercise 12:

$f' = +4cm$ Convergent lens), $oA'_f = F'A'_f - OF' = 24cm - 4cm = 20cm$ (See Figure 2.59).

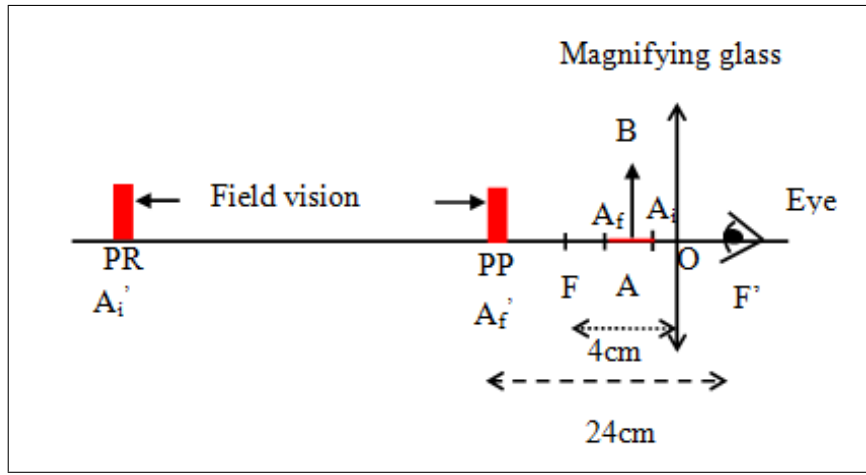


Figure 2.59: Magnifying glass (Latitude).

* Latitude of the magnifying glass verifying:

$$L = \left| \bar{OA}_f - \bar{OA}_i \right|, \quad (2.133)$$

$$= \left| -3.33cm - (-4cm) \right| = 0.66cm. \quad (2.134)$$

where

OA_i : Object position for image at Punctum Remotum (PR).

$\bar{OA}_f$: Object position for image at Punctum Proximum (PP).

with OA_i and $\bar{OA}_f$ verifying the conjugate formulas of the lens;

$$\frac{1}{\bar{OA}_i} = \frac{1}{\bar{OA}'_i} - \frac{1}{f'}, \quad (2.135)$$

$$= \frac{1}{\bar{\infty}} - \frac{1}{+4cm}, \quad (2.136)$$

$$\bar{OA}_i = -4cm. \quad (2.137)$$

$$\frac{1}{\bar{OA}_f} = \frac{1}{-20} - \frac{1}{+4cm}, \quad (2.138)$$

$$\bar{OA}_f = -3.33cm. \quad (2.139)$$

Exercise 13:

Microscope: $\left\{ \begin{array}{l} \text{Eyepiece } (L_{C_1}) \text{ with center } O_1 : f_1 = 4mm, \\ \text{Objective } (L_{C_2}) \text{ with center } O_2 : f_2 = 12mm, \\ \text{The distance between the two lenses } \bar{O_1O_2} = +16cm = +160mm. \end{array} \right.$

Normal eye: $PP = -14cm = -140mm$, $PR \rightarrow -\infty$, The eye on F'_2 .

For $L_{C_1} - L_{C_2}$:

$$AB \text{ (Object } L_{C_1}) \xrightarrow[\substack{\text{Image for } L_{C_1} \\ \text{Object for } L_{C_2}}]{L_{C_1}} A_1B_1 \xrightarrow{L_{C_2}}_{O_2} A'B' \text{ (Image finale).}$$

1/ Position of the object (AB) so that the final image ($A'B'$) is formed at infinity:

$$A_iB_i \xrightarrow{L_{C_1}}_{O_1} A_1B_1 \xrightarrow{L_{C_2}}_{O_2} A'_iB'_i. \quad (2.140)$$

$$\text{For } L_{C_1} \left\{ \begin{array}{l} \text{Object position: } \bar{O_1A_i} \\ \text{Image position: } \bar{O_1A_1} \end{array} \right., \quad \text{For } L_{C_2} \left\{ \begin{array}{l} \text{Object position: } \bar{O_2A_1} \\ \text{Image position: } \bar{O_2A'_i} \end{array} \right.,$$

* For L_{C_2} :

The final image ($A'B'$) is formed at infinity $\left(\bar{O_2A'_i} \rightarrow -\infty \right)$, The object must be on the object focal point of L_{C_2} ($\bar{O_2A_1} = \bar{O_2F_2}$) (See Figure 2.60)

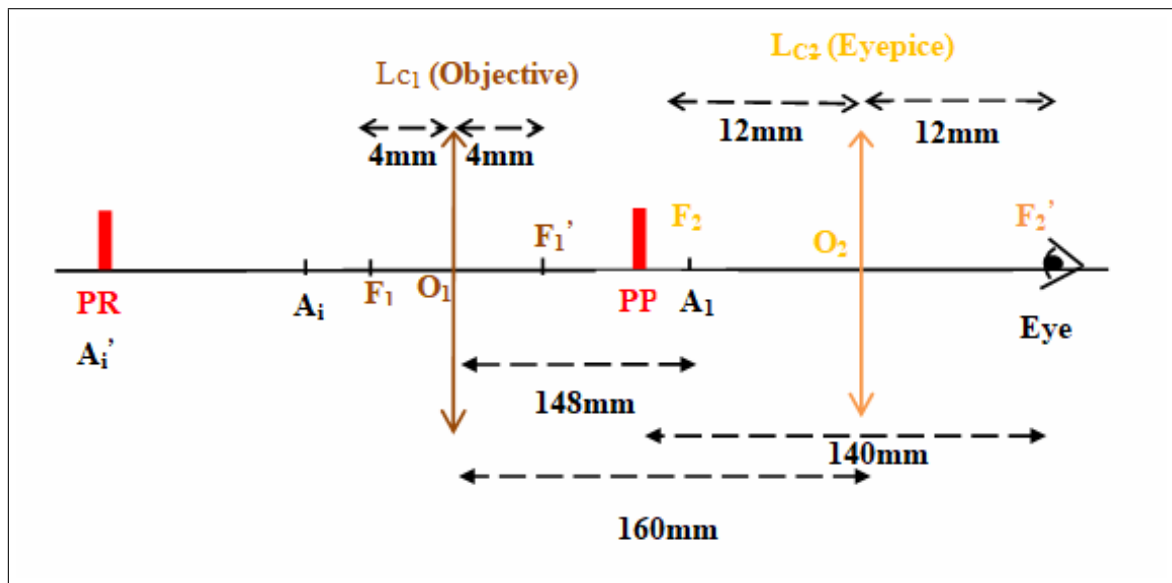


Figure 2.60: Position of the object so that the final image is formed at infinity by microscope.

$$\bar{O}_2 A_1 = -12mm, \quad A_1 \bar{O}_2 = -\bar{O}_2 A_1 = +12mm. \quad (2.141)$$

* * For L_{C_1}

$$\bar{O}_1 O_2 = \bar{O}_1 A_1 + A_1 \bar{O}_2, \quad (2.142)$$

$$\bar{O}_1 A_1 = \bar{O}_1 O_2 - A_1 \bar{O}_2, \quad (2.143)$$

$$= 160mm - (+12mm) = 148mm, \quad (2.144)$$

** For L_{C_2}

$\bar{O}_2 A'_i \rightarrow \infty$, the object must be at the object focal point of the eyepiece lens (L_2);

$$\bar{O}_2 A_1 = \bar{O}_2 F_2 = -12mm, \quad (2.145)$$

$$A_1 \bar{O}_2 = +12mm. \quad (2.146)$$

** For L_{C_1} :

$$\bar{O}_1 O_2 = \bar{O}_1 A_1 + A_1 \bar{O}_2, \quad (2.147)$$

$$\bar{O}_1 A_1 = \bar{O}_1 O_2 - A_1 \bar{O}_2, \quad (2.148)$$

$$= (+160mm) - (+12mm) = +148mm.$$

Conjugate relation for L_{C_1} :

$$\frac{1}{\overline{O_1 A_1}} - \frac{1}{\overline{O_1 A_i}} = \frac{1}{+4mm} \quad (2.149)$$

$$\frac{1}{+148mm} - \frac{1}{\overline{O_1 A_i}} = \frac{1}{+4mm} \quad (2.150)$$

$$\overline{O_1 A_i} = -4.11mm. \quad (2.151)$$

2/ Position of the object (AB) so that the final image ($A'B'$) is formed at $PP = -14cm$ (See Figure 2.61):

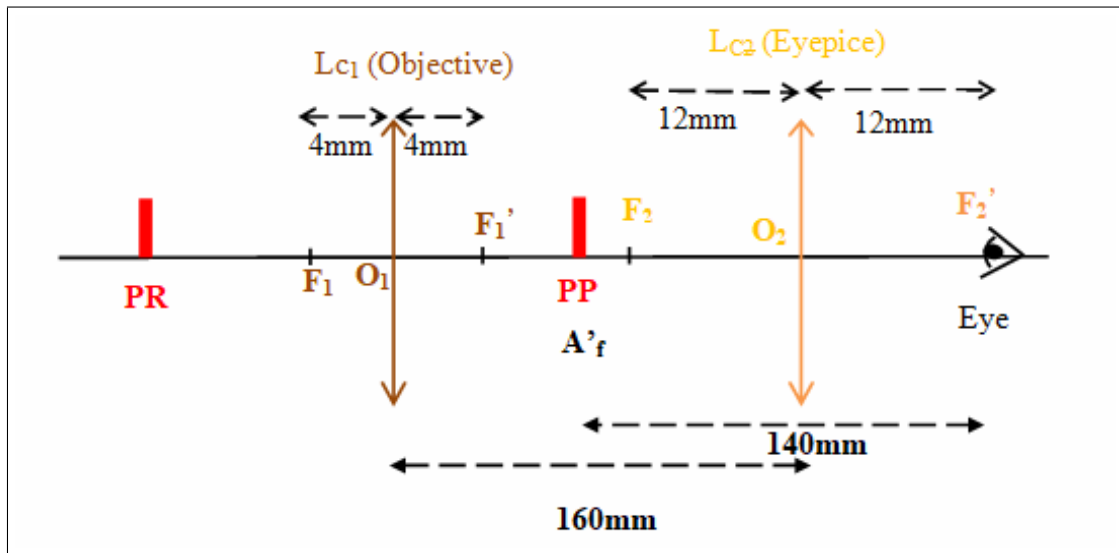


Figure 2.61: Position of the object so that the final image is formed at PP by microscope.

$$A_f B_f \xrightarrow[L_{C_1}]{O_1} A_1 B_1 \xrightarrow[L_{C_2}]{O_2} A'_i B'_i. \quad (2.152)$$

$$\text{For } L_{C_1} \left\{ \begin{array}{l} \text{Object position: } \overline{O_1 A_f} \\ \text{Image position: } \overline{O_1 A_1} \end{array} \right., \quad \text{for } L_{C_2} \left\{ \begin{array}{l} \text{Object position: } \overline{O_2 A_1} \\ \text{Image position: } \overline{O_2 A'_f} \end{array} \right.,$$

* For L_{C_2}

$$\overline{O_2 A'_f} = PP - F'_2 O_2 = (-140mm) - (+12mm), \quad (2.153)$$

$$= -128mm. \quad (2.154)$$

Conjugate relation for L_{C_2} :

$$\frac{1}{\overline{O_2 A'_f}} - \frac{1}{\overline{O_2 A_1}} = \frac{1}{f'_2}, \quad (2.155)$$

$$\frac{1}{-128mm} - \frac{1}{\overline{O_2 A_1}} = \frac{1}{+12mm}. \quad (2.156)$$

$$\overline{O_2 A_1} = -1.19mm. \quad (2.157)$$

** For L_{C_1} :

$$\overline{O_1 O_2} = \overline{O_1 A_1} + \overline{A_1 O_2}, \quad (2.158)$$

$$\overline{O_1 A_1} = \overline{O_1 O_2} - \overline{A_1 O_2}, \quad (2.159)$$

$$= (+160mm) - (-1.19mm) = +161.19mm. \quad (2.160)$$

Conjugate relation for L_{C_1} :

$$\frac{1}{\overline{O_1 A_1}} - \frac{1}{\overline{O_1 A_f}} = \frac{1}{+4mm}, \quad (2.161)$$

$$\frac{1}{+161.19mm} - \frac{1}{\overline{O_1 A_f}} = \frac{1}{+4mm}, \quad (2.162)$$

$$\overline{O_1 A_f} = -4.10mm. \quad (2.163)$$

3/ The focusing latitude of the microscope (L):

$$L = \left| \overline{O_1 A_f} - \overline{O_1 A_i} \right|, \quad (2.164)$$

$$= |(-4.10mm) - (-4.11mm)|, \quad (2.165)$$

$$= 0.01mm. \quad (2.166)$$

Chapter 3

Fluid mechanics

3.1 Introduction

In this chapter, we will begin by introducing fundamental concepts and properties of fluids. Subsequently, we will explore the study of fluids in both static and dynamic states, presenting the basic governing laws along with some simple applications.

3.2 Definition and characteristics of a fluid

3.2.1 Definitions:

- * Fluid mechanics is the study of the behavior of fluids at rest and in motion.

- * Fluids are a group of gases and liquids.

Examples:

Liquids; water, oil, gasoline, etc.

Gases; oxygen, air, etc.

- * A fluid is defined as a continuous body without rigidity (unlike solids) and which can undergo large deformations under the action of forces.

- * Fluid is a deformed body.

- * Fluids are substances that can flow.

Concept of compressibility and incompressibility

* A fluid is said to be compressible if its density ρ varies with pressure or temperature. In general, gases are compressible fluids (ρ varies) and occupy all the volume available to them.

* A fluid is said to be incompressible if its density ρ is constant (ρ does not vary with pressure or temperature).

Notion of viscosity

Viscosity is a quantity which characterizes the internal friction of the fluid, in other words its capacity to flow.

Perfect fluid

A fluid is said to be perfect if it is possible to describe its motion without taking into account the effects of friction.

Real fluid

Unlike a perfect fluid, in a real fluid the internal friction forces that oppose the relative sliding of the fluid layers are taken into consideration.

3.2.2 Characteristics of a fluid**Properties of a fluid**

A fluid is characterized by three main quantities:

Temperature (T):

The temperature is a measure of the internal energy level of a fluid.

Pressure (P):

* **Atmospheric Pressure (P_0):**

It is pressure of the earth's atmosphere. This changes with weather and elevation. Normal atmospheric pressure at sea level (an average value)

* **Absolute Pressure:**

Absolute pressure is always greater than or equal to zero.

* **Gauge Pressure:**

Gauge pressure = absolute pressure – atmospheric pressure

Density (ρ)

Physical characteristics

Density: The density ρ of a liquid given by;

$$\rho = \frac{m}{V}, \quad (3.1)$$

with

ρ : Density in Kg/m^3 .

m : Mass of a fluid in Kg .

V : Volume of a fluid in m^3 .

Volumetric Weight The volumetric weight $\bar{\omega}$ of a fluid is given by:

$$\bar{\omega} = \frac{mg}{V}, \quad (3.2)$$

where

$\bar{\omega}$: Volumetric weight in N/m^3 .

m : Mass of a fluid in Kg .

V : Volume of a fluid in m^3

g : Acceleration due to gravity in m/s^2 .

Specific gravity (relative density): The relative density d of a fluid given by;

$$d = \frac{\rho}{\rho_{ref}} = \frac{\text{density of a fluid}}{\text{density of a reference fluid}}. \quad (3.3)$$

Note • In the case of liquids, **water** will be taken as the **reference fluid**.

• In the case of gases, **air** will be taken as the **reference fluid**.

3.3 Hydrostatics

Fluid **statics** is the branch of fluid mechanics that investigates the properties of fluids at **rest**.

3.3.1 Pressure

Given a force F exerted uniformly and perpendicularly on a flat surface of area S , the pressure P is defined by the relation (See Figure 3.1):

$$P = \frac{F}{S}, \quad (3.4)$$

where:

P : The pressure, measured in **Pascals** (Pa).

F : The force, measured in **Newtons** (N).

S : The surface area, measured in **square meters** (m^2).

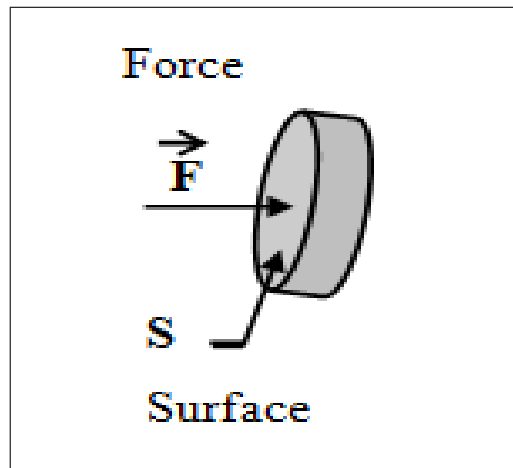


Figure 3.1: Pressur.

Some pressure units:

$$1bar = 10^5 pa, \quad (3.5)$$

$$1Atm = 1,01325 \times 10^5 Pa, \quad (3.6)$$

$$1Atm = 760mmHg. \quad (3.7)$$

Absolute pressure and relative pressure:

Absolute pressure : Absolute pressure is the pressure measured relative to a perfect vacuum (zero pressure). It represents the total pressure exerted by a fluid or gas

Relative pressure (Gauge pressure) : Relative pressure, also known as gauge pressure, is the pressure measured relative to the ambient atmospheric pressure. It can be positive, indicating a pressure higher than atmospheric pressure, or negative, indicating a pressure lower than atmospheric pressure (a vacuum relative to the atmosphere)

Hydrostatic pressure in a homogeneous fluid:

The hydrostatic pressure (P_m) at a depth (h) in a homogeneous fluid is given by the equation;

$$P_m = P_{atm} + \rho gh, \quad (3.8)$$

if we neglect the influence of atmospheric pressure ($P_{atm} = 0$);

$$P_m = \rho gh, \quad (3.9)$$

where

P : The hydrostatic pressure, measured in Pascals (Pa).

ρ : The density of the fluid (mass per unit volume), measured in kilograms per cubic meter (kg/m^3).

h : The depth from the surface of the fluid, measured in meters (m).

g : The acceleration due to gravity, measured in meters per second squared (m/s^2).

3.3.2 Fundamental principle of hydrostatics

The fundamental principle of hydrostatics states that the pressure difference between two points within a fluid at equilibrium is directly proportional to the vertical distance between those points. This relationship is expressed by the equation (See Figure 3.2):

$$P_b - P_a = \rho g(z_b - z_a), \quad (3.10)$$

where

P_b : The pressure at point b .

P_a : The pressure at point a .

ρ : The density of the fluid.

g : The acceleration due to gravity.

z_b : The vertical position of point b .

z_a : The vertical position of point a .

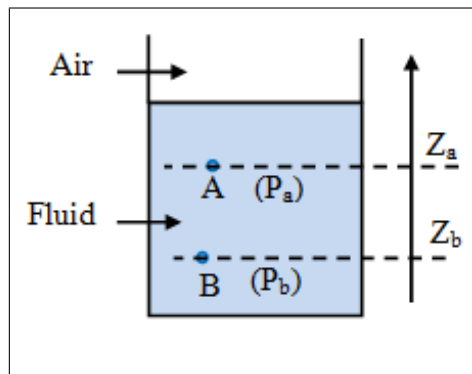


Figure 3.2: Fundamental principle of hydrostatics.

Note

As a consequence, if $z_b = z_a$, then $P_b = P_a$. This implies that the **hydrostatic pressure** is **constant** at all points within the **same horizontal plane** of the fluid (Figure 3.3).

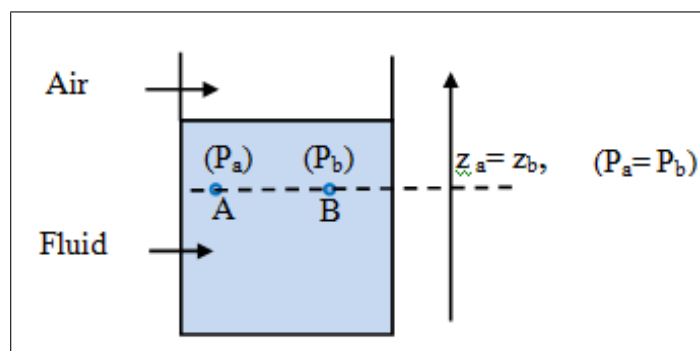


Figure 3.3: Pressure at the same horizontal plane.

3.3.3 Pascal's Theorem (Consequence of the Principle of Hydrostatics):

Pascal's theorem, a direct consequence of the fundamental principle of hydrostatics, states that a pressure change applied to an incompressible fluid in a closed container is transmitted undiminished to every point of the fluid and to the walls of the container.

3.3.4 Equilibrium of immiscible liquids in a container:

When multiple immiscible liquids (liquids that do not mix) are placed in the same container and allowed to reach equilibrium, they will arrange themselves in layers based on their densities.

The liquid with the highest density will settle at the bottom of the container, while the liquid with the lowest density will float on top (See Figure 3.4).

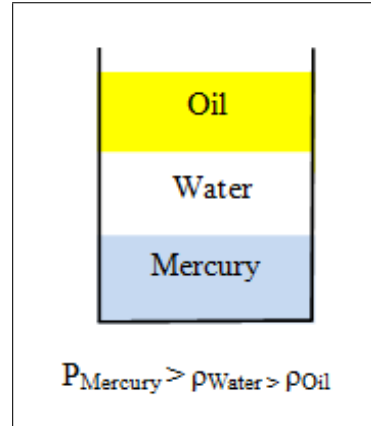


Figure 3.4: Equilibrium of immiscible liquids in a container.

3.3.5 Equilibrium of immiscible liquids in communicating vessels

Taking to liquids in communicating vessels as in the Figure 3.5:

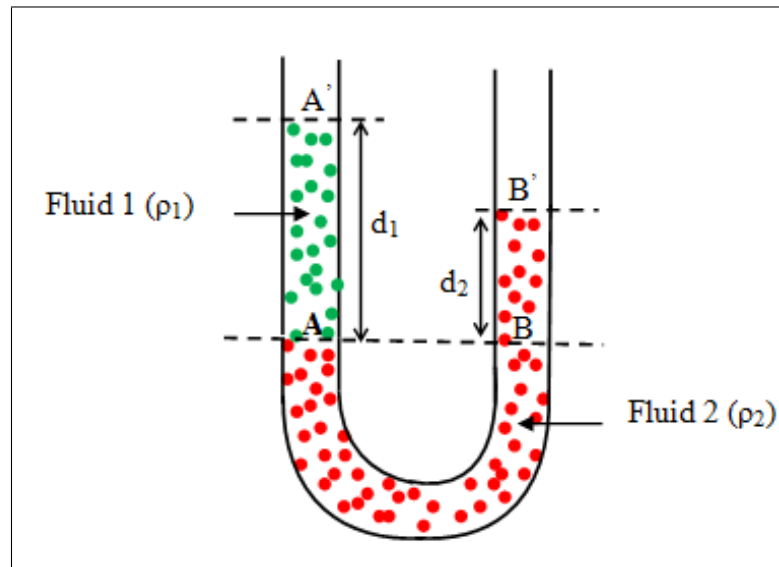


Figure 3.5: Equilibrium of immiscible liquids in communicating vessels.

- In liquid 2, points A and B are at the same depth:

$$P_A = P_B, \quad (3.11)$$

- Atmospheric pressure;

$$P_{A'} = P_{B'}, \quad (3.12)$$

so

$$P_{A'} - P_A = P_B - P_{B'}, \quad (3.13)$$

$$\rho_1 g d_1 = \rho_2 g d_2, \quad (3.14)$$

$$\rho_1 d_1 = \rho_2 d_2. \quad (3.15)$$

3.3.6 Liquid barometer (Torricelli experiment)

Torricelli's experiment is illustrated in the figure below (See Figure 3.6):

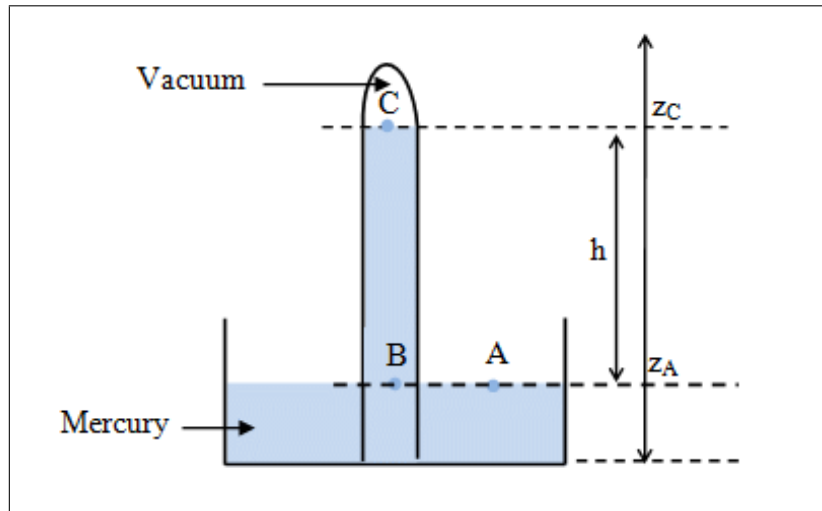


Figure 3.6: Torricelli experiment.

Pressure in the vacuum at the top of the tube

$$P_c = 0, \quad (3.16)$$

Atmospheric pressure at the liquid surface

$$P_B = P_A = P_{atm}, \quad (3.17)$$

$$P_B = \rho g z_b, \quad (3.18)$$

$$P_B = P_c + \rho g (z_c - z_A), \quad (3.19)$$

$$P_B = \rho g h. \quad (3.20)$$

3.3.7 Hydraulic press

It consists of two communicating cylinders of section S_A and S_B (See Figure 3.7):



Figure 3.7: Hydraulic press.

At equilibrium, we have:

$$P_A = P_B, \quad (3.21)$$

$$\frac{F_A}{S_A} = \frac{F_B}{S_B}. \quad (3.22)$$

Hydraulic presses use this principle to multiply force, allowing for heavy lifting and pressing applications.

3.3.8 Archimedes' thrust

An object immersed in a fluid experiences an upward force equal to the weight of the displaced fluid (See Figure 3.8):

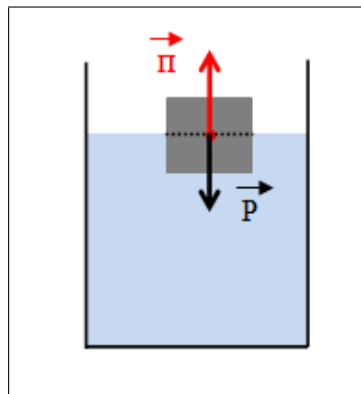


Figure 3.8: Archimedes' thrust.

$$\Pi = \rho_l V_e g, \quad (3.23)$$

$$P = mg = \rho_c V_c g, \quad (3.24)$$

with

Π : Archimedes' thrust.

P : Weight of the object.

ρ_l : Density of the liquid.

ρ_c : Density of the object.

V_c : Volume of the object.

V_e : Volume of the immersed body in the fluid.

g : Acceleration of gravity in m/s^2 .

Conditions:

- For $P > \Pi$, $\rho_c > \rho_l$, the object **sinks**.
- For $P = \Pi$, $\rho_c = \rho_l$, the object is in **equilibrium**.
- For $P < \Pi$, $\rho_c < \rho_l$, the object **floats**.

3.4 Hydrodynamics

3.4.1 Definitions

Hydrodynamics is the study of the relationships between molecular forces and the **movements** of fluids.

Velocity

During fluid flow, each particle of matter, considered as a point, has a velocity V at each instant and describes, as time varies, a curve called a trajectory or fluid line.

Note

The velocity can:

- Vary from one point in the fluid to another.
- Vary with time at each point.

Streamlines

Streamlines are lines that are tangent at each of their points to the direction of the flow velocity vectors at instant (t) (See Figure 3.9).

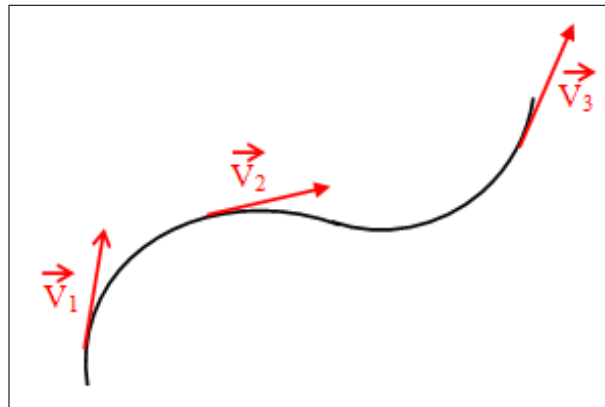


Figure 3.9: Streamlines.

Stream tube - Filament - Vein

A flow tube represents an elementary portion of the fluid in motion, limited by streamlines (See Figure 3.10).

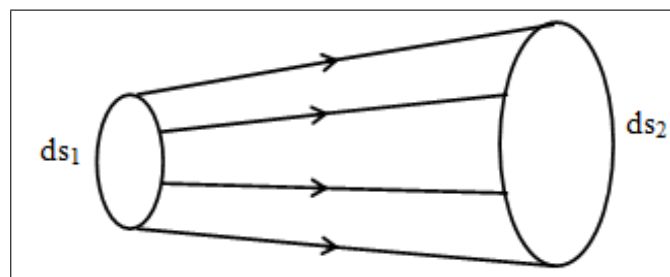


Figure 3.10: Stream tube.

Notes

- 1/ A **flow filament** or fluid filament is a flow tube of infinitely small cross-section.
- 2/ The juxtaposition of an infinite number of fluid filaments or several flow tubes gives a fluid **vein**.
- 3/ A **stream** tube represents an elementary part of the fluid in motion, limited by current lines.

Steady Flow (Permanent Flow)

* A flow is steady or permanent when the velocity field does not depend on time.

* Such a movement also implies that pressure, temperature, and density do not depend on time.

* In this movement, velocities vary only from one point in the fluid to another, but at each point, the velocity always has the same value regardless of time.

* The velocity is a function of the coordinates x, y, z of the point; $v = f(x, y, z)$.

3.4.2 Flow rate

The movement of an ideal liquid in a pipe of cross-section S is characterized by its **flow rate** Q .

Mass flow rate:

If Δm is the mass of fluid that has crossed a right section of the pipe during time Δt , by definition, the mass flow rate is:

$$Q_m = \frac{\Delta m}{\Delta t}, \quad (3.25)$$

with

Δm : In Kg .

Δt : In s .

Q : In $Kg.s^{-1}$.

Volume flow rate:

If ΔV is the volume of fluid that has crossed a right section of the pipe during time Δt , by definition, the volume flow rate is:

$$Q_v = \frac{\Delta V}{\Delta t}, \quad (3.26)$$

with

ΔV : In m^3 .

Δt : In s .

$$Q : \text{In } m^3.s^{-1}.$$

Note:

If S , the cross-section of the tube, is **constant**. The liquid moves a distance Δx during time Δt (See Figure 3.11).

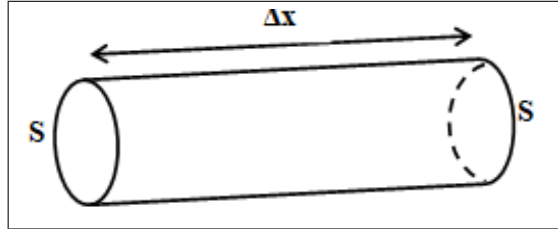


Figure 3.11: Volume of fluid.

The volume of liquid exiting the tube is:

$$\Delta V = S.\Delta x = S.v.\Delta t, \quad (3.27)$$

and since

$$\Delta V = Q_v.\Delta t, \quad (3.28)$$

therefore

$$Q_v = S.v. \quad (3.29)$$

By using the equations relating the mass flow rate, the volumetric flow rate, and the density;

$$\rho = \frac{\Delta m}{\Delta V}, \quad (3.30)$$

one finds that

$$Q_m = \rho.Q_v. \quad (3.31)$$

3.4.3 Basic principles:

The mathematical analysis of fluid flow can be carried out using the following basic principles:

- 1/ The principle of mass conservation, from which the continuity equation is established.
- 2/ The principle of energy conservation, which allows obtaining the fundamental equation of fluid flow.

3.4.4 Continuity equation

In a circulatory circuit of a fluid whose cross-section is variable from one place to another, the circulatory velocity v is also variable, but the flow rate must remain constant (if it is a rigid and non-deformable tube), following the principle that the quantity of liquid that enters one end of the circuit must exit the other end (See Figure 3.12).

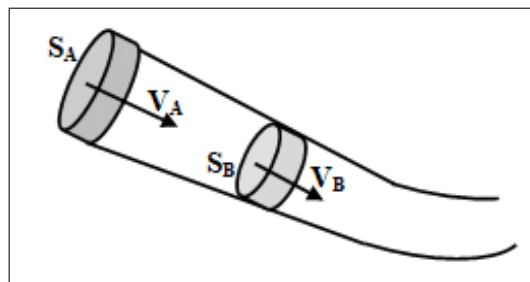


Figure 3.12: Continuity equation.

For a closed surface; entering mass flow rate = exiting mass flow rate,

$$Q_{m_A} = Q_{m_B}, \quad (3.32)$$

$$\frac{\Delta m_A}{\Delta t_A} = \frac{\Delta m_B}{\Delta t_B}, \quad (3.33)$$

$$\frac{\rho \cdot \Delta V_A}{\Delta t_A} = \frac{\rho \cdot \Delta V_B}{\Delta t_B}, \quad (3.34)$$

$$Q_v = S_A \cdot v_A = S_B \cdot v_B = cte \quad . \quad (3.35)$$

3.4.5 The mechanical energy of a fluid

A moving liquid possesses three forms of mechanical energy linked respectively to pressure, altitude, and velocity. For the first two, it is potential energy, and for the third, kinetic energy.

These forms of energy are generally expressed in units of pressure (it is in fact the energy per unit volume; $J.m^{-3} = Pa$).

Energy linked to **pressure** (P):

$$E_{p_1} = P. \quad (3.36)$$

Energy linked to **altitude** (h):

$$E_{p_2} = \rho gh. \quad (3.37)$$

Flow energy of the circulatory **velocity** (v):

$$E_{p_3} = 1/2\rho v^2. \quad (3.38)$$

The **total mechanical energy** (E_m) of the fluid is then the sum of these three terms:

$$E_m = P + \rho gh + 1/2\rho v^2. \quad (3.39)$$

3.4.6 Bernoulli's theorem:

This theorem simply expresses that the total mechanical energy of an ideal fluid (without head loss) is constant in a circuit in which it circulates at constant flow rate (over time) (See Figure 3.13):

$$E_m = P + \rho gh + 1/2\rho v^2 = cte. \quad (3.40)$$

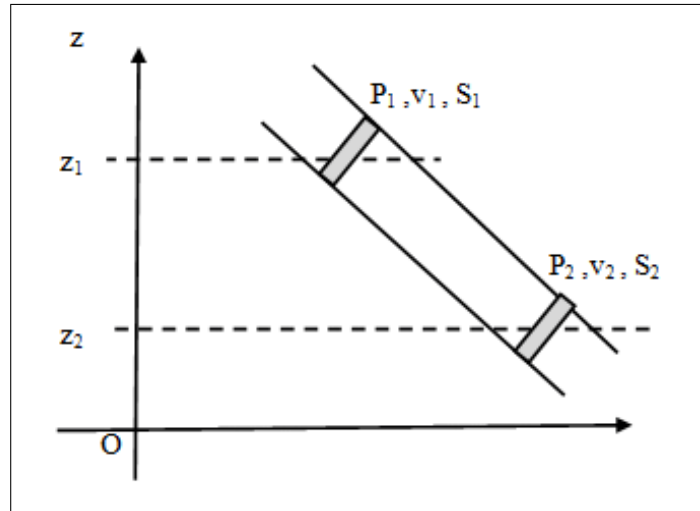


Figure 3.13: Bernoulli's theorem.

Notes:

1/ For a static fluid: $v = 0$, **Bernoulli's theorem** is reduced to;

$$P + \rho gh = cte. \quad (3.41)$$

2/ The different forms of energy (potential and kinetic) can, on the other hand, transform into one another, provided that the total energy remains constant.

$$P_1 + \rho g z_1 + 1/2\rho v_1^2 = P_2 + \rho g z_2 + 1/2\rho v_2^2. \quad (3.42)$$

Conditions for Applying Bernoulli's Theorem:

The conditions for applying Bernoulli's theorem are:

- Incompressible fluid with constant density.
- Non-viscous fluid (no friction), therefore no head loss (no pressure loss).
- Fluid in steady (constant flow velocity) and non-turbulent flow.

3.4.7 Application of Bernoulli's Theorem

Let's examine the application of this theorem to three common devices: the Torricelli's vase, the Venturi effect, and the Pitot tube.

Torricelli's Vase (Torricelli's Theorem)

Consider a cylindrical tank filled with a liquid, in which a small orifice is opened near the bottom (See Figure 3.14).

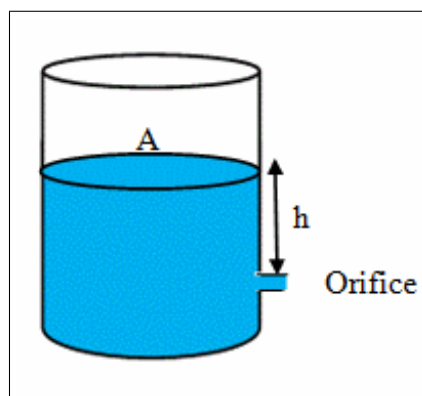


Figure 3.14: Torricelli's Vase.

Torricelli's formula relates the efflux velocity of the liquid to the height of the liquid above the orifice.

Analysis using Bernoulli's Theorem:

Consider two points:

Point A: at the free surface of the liquid inside the tank.

Point B: at the exit of the orifice.

We can make the following assumptions:

The pressure at the free surface and the orifice exit is atmospheric pressure:

$$P_A = P_B = P_{atm}. \quad (3.43)$$

The volumetric flow rate Q is constant

$$Q = v_A S_A = v_B S_B, \quad (3.44)$$

where S_A is the cross-sectional area of the tank and S_B is the cross-sectional area of the orifice.

Since the tank's cross-sectional area is much larger than the orifice's ($S_A \gg S_B$), the velocity at the free surface is negligible compared to the velocity at the orifice: $v_A \ll v_B$, which implies

$$\frac{S_A}{S_B} = \frac{v_B}{v_A} \gg 1. \quad (3.45)$$

Therefore, we can approximate $v_A \approx 0$.

The height difference between points A and B is

$$h = z_A - z_B. \quad (3.46)$$

Applying Bernoulli's equation between points A and B :

$$P_A + \rho g z_A + 1/2 \rho v_A^2 = P_B + \rho g z_B + 1/2 \rho v_B^2. \quad (3.47)$$

Simplifying the equation and Solving for v_B , we obtain **Torricelli's formula**:

$$v_B = \sqrt{2gh}. \quad (3.48)$$

This formula shows that the efflux velocity of the liquid through the orifice is the same as the speed an object would acquire falling freely from a height h .

Venturi effect

When an incompressible fluid flows through a constricted section of a pipe (a Venturi), its velocity increases, and its static pressure decreases. This is known as the Venturi effect (Figure 3.15).

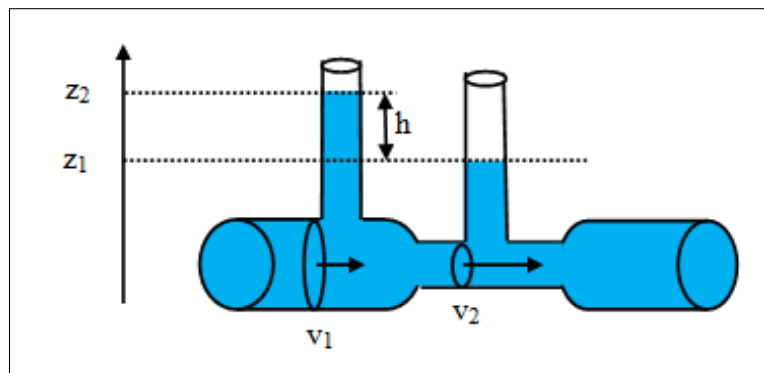


Figure 3.15: Venturi effect.

Analysis using Bernoulli's Theorem:

Consider a horizontal pipe ($z_1 - z_2 = z$) with two different cross-sectional areas, S_1 and S_2 ($S_1 > S_2$), and two corresponding points (1 and 2) along the same streamline

Bernoulli's equation for a horizontal pipe becomes:

$$P_1 + 1/2\rho v_1^2 = P_2 + 1/2\rho v_2^2, \quad (3.49)$$

The continuity equation for an incompressible fluid states that the volumetric flow rate is constant:

$$Q = v_1 S_1 = v_2 S_2, \quad (3.50)$$

since $S_1 > S_2$, it follows that $v_1 > v_2$.

Rearranging Bernoulli's equation to find the pressure difference:

$$\Delta P = P_1 - P_2 = 1/2\rho (v_2^2 - v_1^2), \quad (3.51)$$

substituting the expression for v_2 in terms of v_1 and the areas:

$$v_1 = \sqrt{\frac{2\Delta P}{\rho \left(\left(\frac{S_1}{S_2} \right)^2 - 1 \right)}}, \quad (3.52)$$

The value of the velocity v_1 can be deduced from the pressure difference measured by a manometer:

Pitot tube

A Pitot tube is used to measure the flow speed of a fluid. It typically consists of two tubes immersed in the fluid flow: one tube (opening at point A) facing directly into the flow to measure the stagnation pressure, and another tube (opening at point B) oriented parallel to the flow to measure the static pressure. The openings of both tubes are at the same height (See Figure 3.16).

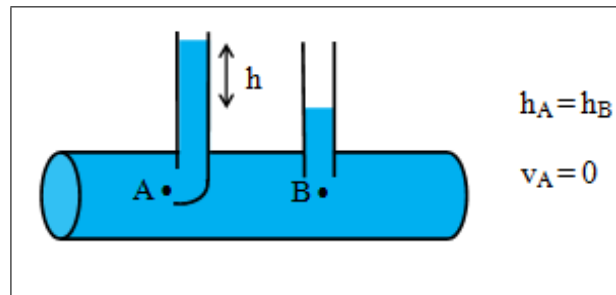


Figure 3.16: Pitot tube.

Analysis using **Bernoulli's Theorem**:

Consider two points:

Point A : the stagnation point at the opening of the Pitot tube facing the flow. At this point, the fluid velocity is zero ($v_A = 0$), and the pressure is the stagnation pressure P_A .

Point B : a point in the undisturbed flow stream, at the same height as point A ($z_A = z_B$), where the fluid velocity is $v_B = v$ (the velocity to be measured) and the pressure is the static pressure

$$P_B = P. \quad (3.53)$$

Applying Bernoulli's theorem between points A and B (since $z_A = z_B$):

$$P_A = P_B + \frac{1}{2}\rho v_B^2. \quad (3.54)$$

$$P_A = P_0 + \rho g z_A. \quad (3.55)$$

The difference between the stagnation pressure (P_A) and the static pressure (P_B) is the dynamic pressure:

$$\Delta P = P_A - P_B = \frac{1}{2}\rho v^2. \quad (3.56)$$

By measuring this pressure difference (e.g., using a manometer connected to the two tubes), the flow velocity v can be determined:

$$v = \sqrt{\frac{2\Delta P}{\rho}}. \quad (3.57)$$

If a differential manometer is used, measuring a height difference h of a manometric fluid with density (ρ), then the pressure difference is

$$\Delta P = P_A - P_B = \rho gh. \quad (3.58)$$

In this case, the flow velocity becomes:

$$v = \sqrt{2gh}. \quad (3.59)$$

Thus, by measuring the height difference h of the liquid in the two tubes, the flow velocity v of the fluid can be deduced.

3.5 Exercises

Exercise 1:

Calculate the absolute pressure in (Pa) at a depth of $6m$ below the free surface of a body of water. Assume: $P_{atm} = 10^5 pa$.

Exercise 2:

Find the absolute pressure and gauge pressure at point A, B and C as shown in the Fig.

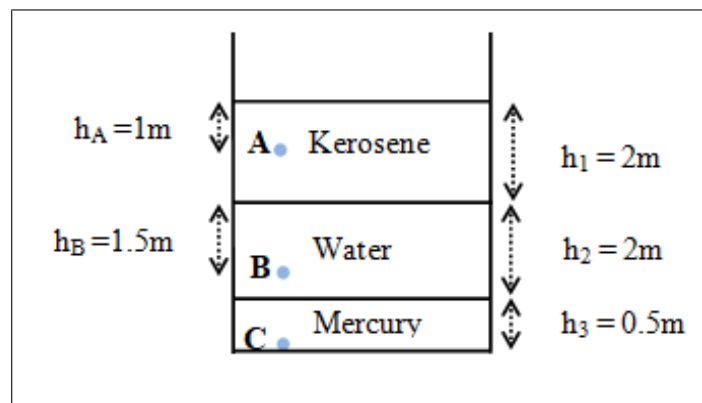


Figure :

where

$1 \text{ atm} = 10^5 \text{ Pa}$, $\rho_1 = 800 \text{ kg/m}^3$ (Kerosene), $\rho_2 = 10^3 \text{ kg/m}^3$ (Water), $\rho_3 = 13.6 \times 10^3 \text{ kg/m}^3$ (Mercury). $g = 10 \text{ m/s}^2$

Exercise 3:

The figure opposite shows an open reservoir, equipped with two piezometric tubes and filled with two immiscible liquids A and B with densities of 0.72 and 2.36 respectively.

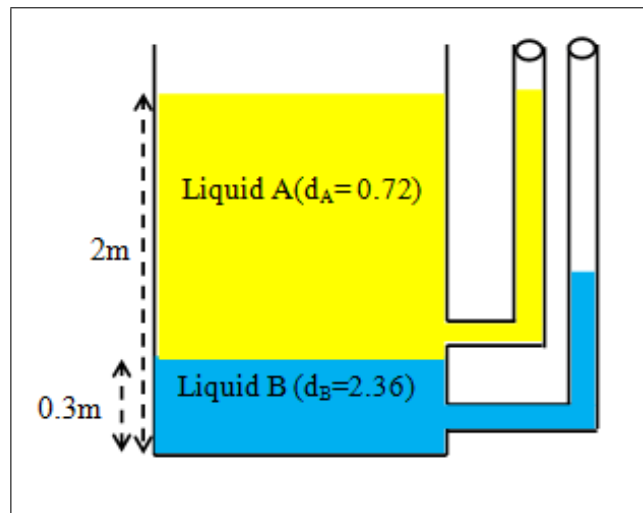


Figure :

Find:

- 1/ The height of the liquid surface in piezometer A ,
- 2/ The height of the liquid surface in piezometer B ,
- 3/ The total pressure at the bottom of the reservoir.

Exercise 4:

Water is flowing through a horizontal tube of non-uniform cross-section. At a place, the radius of the tube is 1.0 cm and the velocity of water is 2 m/s . What will be the velocity of water, where the radius of the pipe is 2.0 cm ?

Exercise 5:

Considering a tank filled with water to a height ($H = 2 \text{ m}$) equipped with a small orifice at its base with a diameter ($d = 10 \text{ mm}$).

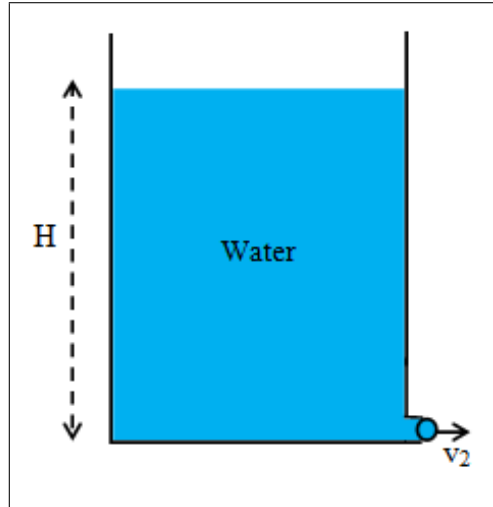


Figure ;

1/ By specifying the assumptions taken into account, apply Bernoulli's theorem to calculate the water flow velocity.

2/ Deduce the volumetric flow rate (Q) in liters per second (L/s) at the outlet of the orifice.

Assume ($g = 9.81 m/s^2$)..

3.6 Solutions

Exercise 1:

1/Absolute pressure (P) in (Pa):

$$P = \rho_e g h + P_{atm} \quad (3.60)$$

$$= 10^3 (kg/m^3) \times 9.81 (m/s^2) \times 6 (m) + 10^5 Pa, \quad (3.61)$$

$$= 58860 pa + 10^5 Pa \quad (3.62)$$

$$= 158860 Pa. \quad (3.63)$$

Exercise 2:

Absolute pressure and gauge pressure at point A, B and C

* Point A

Gauge pressure ($P_{A(gauge)}$):

$$P_{A(gauge)} = \rho_1 g h_A, \quad (3.64)$$

$$= 800 \, (kg/m^3) \times 10 \, (m/s^2) \times 1 \, (m), \quad (3.65)$$

$$= 8kPa. \quad (3.66)$$

Absolute pressure (P_A):

$$P_A = P_{A(gauge)} + P_{atm} \quad (3.67)$$

$$= 8 \times 10^3 \, (Pa) + 10^5 \, (Pa) \quad (3.68)$$

$$= 108kPa. \quad (3.69)$$

*Point B

Gauge pressure ($P_{B(gauge)}$):

$$P_{B(gauge)} = \rho_2 g h_B + \rho_1 g h_1, \quad (3.70)$$

$$= 10^3 \, (kg/m^3) \times 10 \, (m/s^2) \times 1.5 \, (m) + 800 \, (kg/m^3) \times 10 \, (m/s^2) \times 2 \, (m) \quad (3.71)$$

$$= 31kPa. \quad (3.72)$$

Absolute pressure (P_B):

$$P_B = P_{B(gauge)} + P_{atm}, \quad (3.73)$$

$$= 31 \times 10^3 \, (Pa) + 10^5 \, (Pa), \quad (3.74)$$

$$= 131kPa. \quad (3.75)$$

*Point C

Gauge pressure ($P_{C(gauge)}$):

$$P_{C(gauge)} = \rho_3 g h_1 + \rho_2 g h_2 + \rho_1 g h_1, \quad (3.76)$$

$$= 104kPa. \quad (3.77)$$

Absolute pressure (P_C):

$$P_A = P_{A(gauge)} + P_{atm}, \quad (3.78)$$

$$= 104 (kPa) + 100(kPa), \quad (3.79)$$

$$= 204kPa. \quad (3.80)$$

Exercise 3:

* The height of the liquid surface in piezometer A :

Liquid A will rise in piezometer A to the same height as liquid A in the reservoir; $h = 2m$.

* The height of the liquid surface in piezometer B :

Liquid B will rise in piezometer B to the height of $0.3m$ (pressure exerted by liquid B) plus an additional height h' due to the pressure P_A of liquid A :

$$P_A = \rho_A g h_A = d_A \rho_e g h_A = 0.720 \times 1000 \times 9.80 \times (2 - 0.3), \quad (3.81)$$

$$= 11995.2Pa. \quad (3.82)$$

with $\rho_A = d\rho_e$ and $h_A = 2 - 0.3 = 1.7m$.

The additional height h' due to the pressure P_A of liquid

$$P_A = \rho_B g h' = d\rho_e g h', \quad (3.83)$$

$$h' = \frac{P_A}{\rho_B g} = \frac{P_A}{d_B \rho_e g}, \quad (3.84)$$

$$= \frac{11995.2Pa}{2.63 \times 1000 (kg/m^3) \times 9.8 (m/s^2)}, \quad (3.85)$$

$$= 0.465m. \quad (3.86)$$

Liquid B will rise in piezometer B to the height $0.3 + 0.465m = 0.765m$.

The pressure at the bottom:

$$P = \rho_A g h_A + \rho_B g h_B, \quad (3.87)$$

$$= d_A \rho_e g h_A + d_B \rho_e g h_B, \quad (3.88)$$

$$= 0.720 \times 1000 \times 9.80 \times 1.7 + 2.63 \times 1000 \times 9.80 \times 0.3, \quad (3.89)$$

$$= 19727.4Pa, \quad (3.90)$$

$$\simeq 0.19bar. \quad (3.91)$$

Exercise 4:

The velocity of water, where the radius of the pipe is $R_2 = 2.0\text{cm}$;
the section S_2 reads

$$S_2 = \pi R_2^2, \quad (3.92)$$

$$= 3.14 \times (2)^2 \text{ cm}^2, \quad (3.93)$$

$$= 12.56 \text{ cm}^2. \quad (3.94)$$

Using the continuity equation

$$v_2 = \frac{S_1 v_1}{S_2}, \quad (3.95)$$

$$= \frac{1 \text{ (cm}^2\text{)} \times 2 \text{ (m/s)}}{12.56 \text{ (cm}^2\text{)}}, \quad (3.96)$$

$$= 0.16 \text{ m/s}. \quad (3.97)$$

Exercise 5:

1/ Flow Velocity v_2

We apply Bernoulli's theorem between two points: (1) on the free surface of the water and (2) at the orifice.

$$P_1 + \rho g z_1 + 1/2 \rho v_1^2 = P_2 + \rho g z_2 + 1/2 \rho v_2^2. \quad (3.98)$$

This is done with the following assumptions: ($v_1 = 0$) because the water level in the reservoir changes slowly and

$$P_1 = P_2 = P_{atm}, \quad (3.99)$$

the velocity written us

$$v_2 = \sqrt{2gH}, \quad (3.100)$$

$$= \sqrt{2 \times 9.8 \times 2}, \quad (3.101)$$

$$= 6.26 \text{ m/s}. \quad (3.102)$$

2/ Volume flow rate (Q_v)

At first, we calculate the surface of the orifice;

$$S_2 = \pi \times \left(\frac{d}{2}\right)^2, \quad (3.103)$$

$$= 3.14 \times \left(\frac{10}{2}\right)^2, \quad (3.104)$$

$$= 78.5 \text{ mm}^2, \quad (3.105)$$

$$= 78.5 \times 10^{-6} \text{ m}^2. \quad (3.106)$$

The volume flow rate (Q_v) reads

$$Q_v = S_2 \times v_2, \quad (3.107)$$

$$= 6.26 \text{ (m/s)} \times 78.5 \times 10^{-6} \text{ (m}^2\text{)}, \quad (3.108)$$

$$= 491.41 \times 10^{-6} \text{ m}^3/\text{s}, \quad (3.109)$$

$$= 491.41 \times 10^{-6} \times 10^3 \text{ l/s}, \quad (3.110)$$

$$= 0.49 \text{ l/s}. \quad (3.111)$$

where $1 \text{ m}^3 = 10^3 \text{ l}$.

Chapter 4

Concept of crystallography

4.1 Introduction

Matter exists in three states: Gas, Liquid, and Solid. Within the solid state, we can distinguish two main types:

- * **Amorphous solids:** These have a disordered arrangement of their constituent particles.

Examples: Glass or plastic.

- * **Crystalline solids:** These are characterized by a regular and periodic arrangement of their constituent particles.

Examples: Salt and diamonds.

In this chapter, we will introduce the fundamental concepts of crystallography.

4.2 Crystallography

Crystallography is the science dedicated to the study of the forms, structures, and physical properties of crystals at the atomic scale.

4.3 Crystal

A crystal is a solid with a structure composed of atoms arranged in a periodic, and even triperiodic and symmetrical, lattice (Figure 4.1).

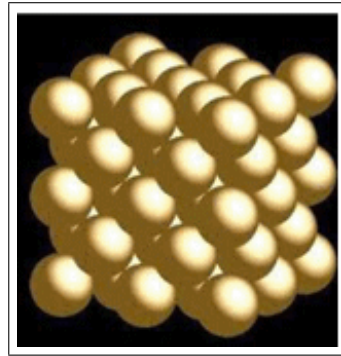


Figure 4.1: Cristal.

4.4 Crystal lattice

A crystal lattice is a set of points or "nodes" arranged periodically in;

* **A direction (one-dimensional):** (Figure 4.2);

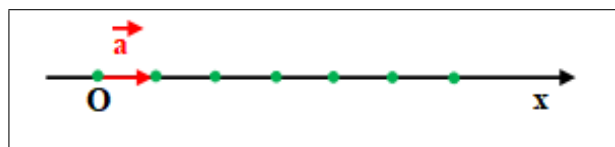


Figure 4.2: One-dimensional lattice

Every lattice point can be generated from an arbitrarily chosen origin through a translational operation defined by:

$$\vec{T} = x\vec{a}, \quad (4.1)$$

where x is an integer, either positive or negative.

* **A plane (two-dimensional):** (Figure 4.3);

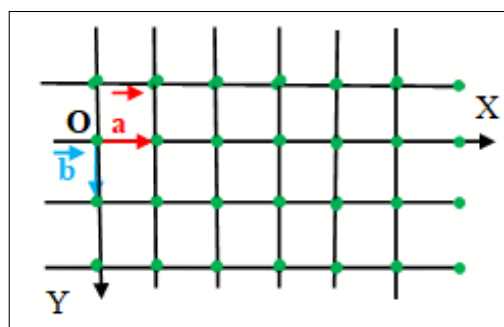


Figure 4.3: Two-dimensional lattice.

The position of any point in this lattice can be generated by a translation vector T expressed as:

$$\vec{T} = x\vec{a} + y\vec{b}, \quad (4.2)$$

where x and y are integers (including positive, negative, and zero).

* **Space (three-dimensional)** (Figure 4.4);

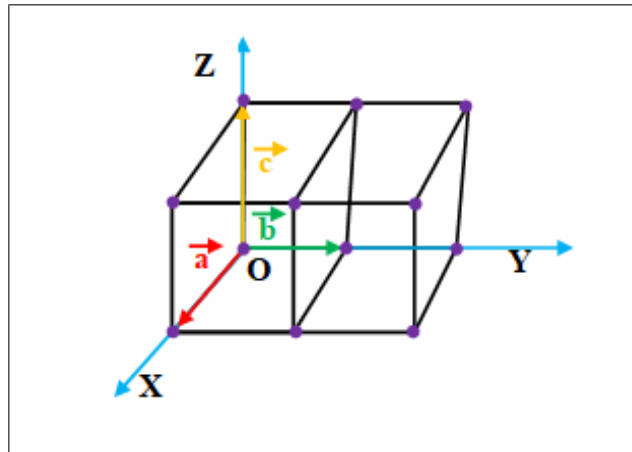


Figure 4.4: Three-dimensional lattice

The entire set of these points (nodes) can be generated from an arbitrary origin node by translations defined by the vector T , such that;

$$\vec{T} = x\vec{a} + y\vec{b} + z\vec{c}, \quad (4.3)$$

where $\vec{a}$, $\vec{b}$ and $\vec{c}$ are three non-coplanar vectors, x , y , and z are the coordinates of a given node, and they are integers (positive, negative, or zero).

4.5 Nodes of a crystal

Nodes are equivalent points in space where motifs are located (Figure 4.5).

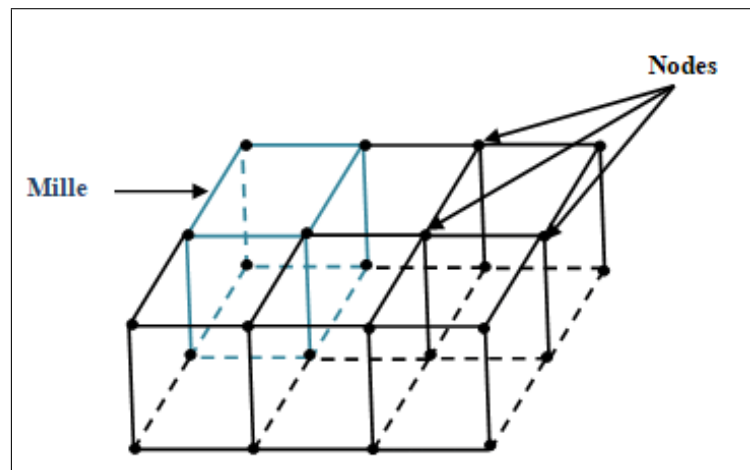


Figure 4.5: Nodes of a crystal.

4.6 Motifs

A motif is a set of particles (atoms, ions, or molecules) regularly distributed in space (Figure 4.6).

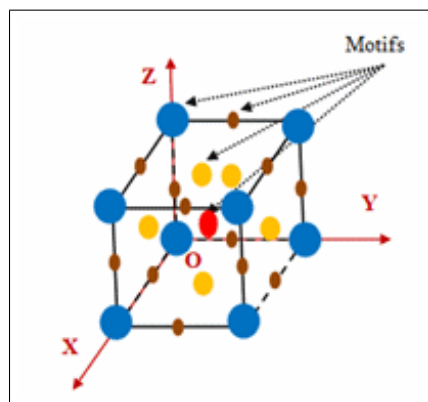


Figure 4.6 : Motifs.

4.7 Crystal structure

Crystal structure = lattice + motif (the base). (Figure 4.7).

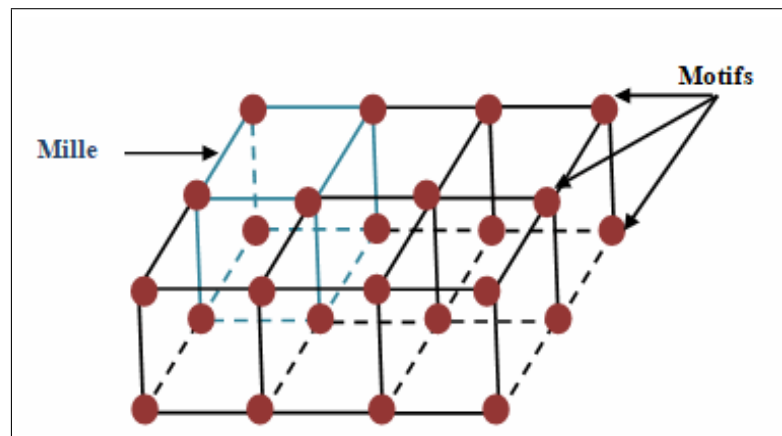


Figure 4.7: Crystal structure.

4.8 Crystalline unit cell

A unit cell is the basic unit from which the entire crystal can be generated.

Example: the unit cell of the Crystal structure (Figure 4.7) is cubic (Figure 4.8).

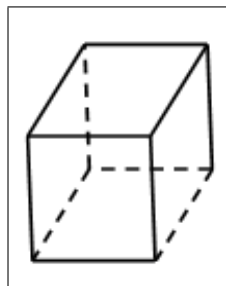


Figure 4.8: unit cell

Within a given crystal, multiple unit cells of different orientations and volumes can be defined. However, the unit cell with the smallest possible volume and the highest possible symmetry is chosen (See Figure 4.9).

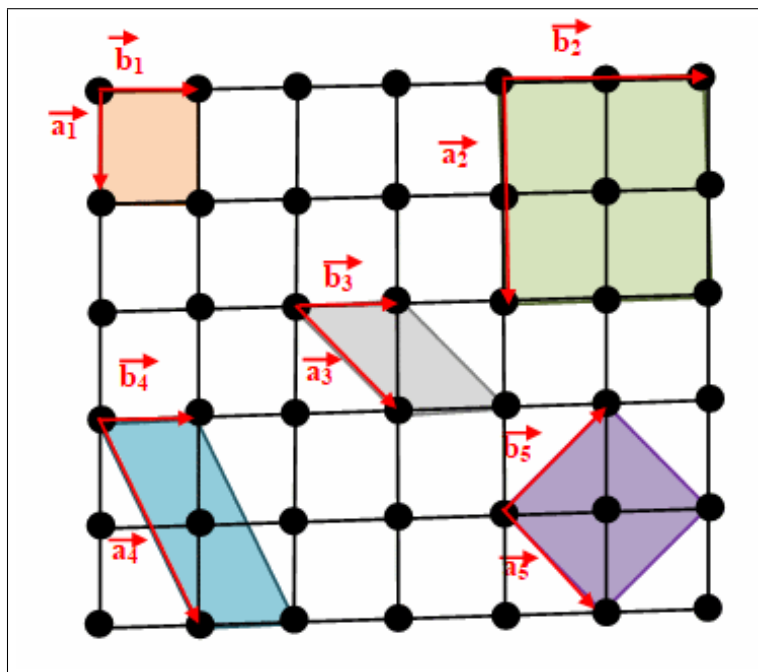


Figure 4.9: Unit cells.

4.9 Lattice Row

A lattice row $[u \ v \ w]$ is any straight line passing through the origin O and the node with coordinates (u, v, w) (Figure 4.).

Notes:

- * The indices u, v, w are coprime (relatively prime); they are the coordinates of the first node closest to the origin. They can be negative, positive, or zero.
- * Negative indices are overlined.
- * When one of the indices u, v or w is negative, the index is written with a bar above it, for example, $[\bar{1}\bar{1}3]$ instead of $[-1 - 13]$.
- * The row $[u \ v \ w]$ contains multiple nodes.
- * The equidistance of the nodes is called the period or parameter of the row.
- * There are multiple identical and equidistant lattice rows that are parallel to it. The set of these rows constitutes a family of lattice rows.
- * The row representing the entire family is the one passing through the origin.
- * The row $[uvw]$ is called any straight line passing through the origin O and the node with coordinates (u, v, w) .

* The unit vector of a reticular row, denoted as $\vec{n}_{uvw}$, is defined as:

$$\vec{n}_{uvw} = u\vec{a} + v\vec{b} + w\vec{c}, \quad (4.4)$$

* The equidistance between consecutive nodes (lattice points) along a reticular row is called the row parameter or period.

* The row parameter (or period), denoted as p , is the magnitude of the unit vector $\vec{n}_{uvw}$:

$$p = |\vec{n}_{uvw}|, \quad (4.5)$$

$$= \sqrt{u^2\vec{a} + v^2\vec{b} + w^2\vec{c} + 2uv\vec{a}\vec{b} + 2uw\vec{a}\vec{c} + 2vw\vec{b}\vec{c}}, \quad (4.6)$$

Examples: Lattice row (Figure 4.10, Figure 4.11);

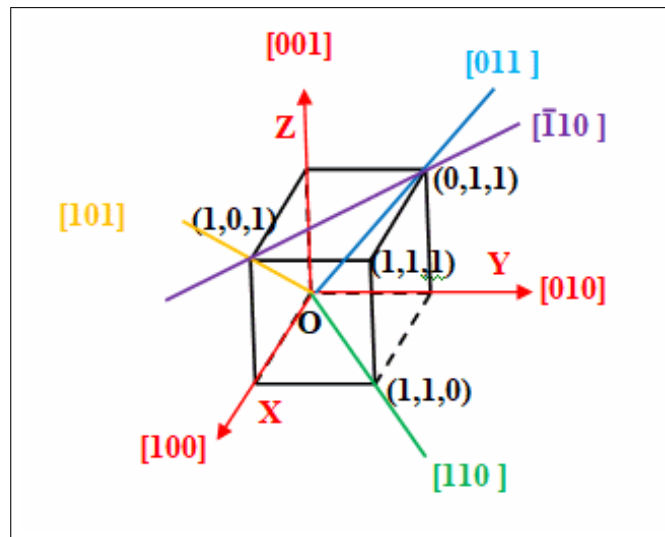


Figure 4.10: Example1(Lattice row)

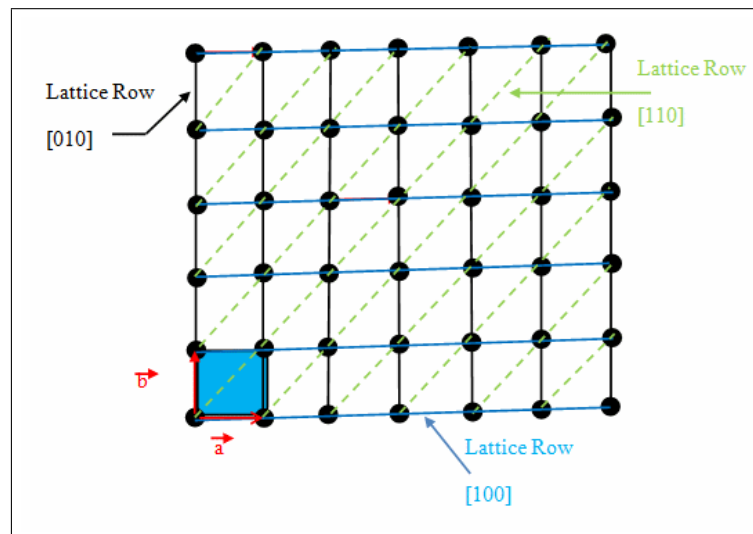


Figure 4.11: Example 2 (Lattice Row)

4.10 Lattice plane

* A plane that passes through the nodes and intersects the x, y, z axes at $\frac{a}{h}, \frac{b}{k}, \frac{c}{l}$ respectively.

* A lattice plane is designated by Miller indices (hkl) , with $h, k, l \in \mathbb{Z}$ (See Figure 4.12).

* The equation of a plane of order m is written:

$$xh + yk + zl = m \quad (4.7)$$

with $m \in \mathbb{Z}$ and m represents the N th plane from the origin.

* There is a family $(h \ k \ l)$ of parallel and equidistant lattice planes.

* The plane representing a given family $(h \ k \ l)$ is the plane closest to the origin and not the one passing through the origin, such that the network plane intersects the crystallographic axes at points A, B and C where:

$$OA = \frac{a}{h}, OB = \frac{b}{k}, OC = \frac{c}{l}. \quad (4.8)$$

* When one of the Miller indices is negative, the index is written with a bar above it.

* The equidistance of planes of the same family is called the interplanar spacing and is denoted d_{hkl} .

* The Miller indices of a plane (a family $(h \ k \ l)$) are determined as follows:

- 1/ The intersection points of this plane with the three axes are found.
- 2/ The reciprocals of the found numbers are taken, and a triplet of integers (usually the smallest) with the same ratio between them as these reciprocals is taken.
- 3/ The result is placed in parentheses $(h \ k \ l)$.

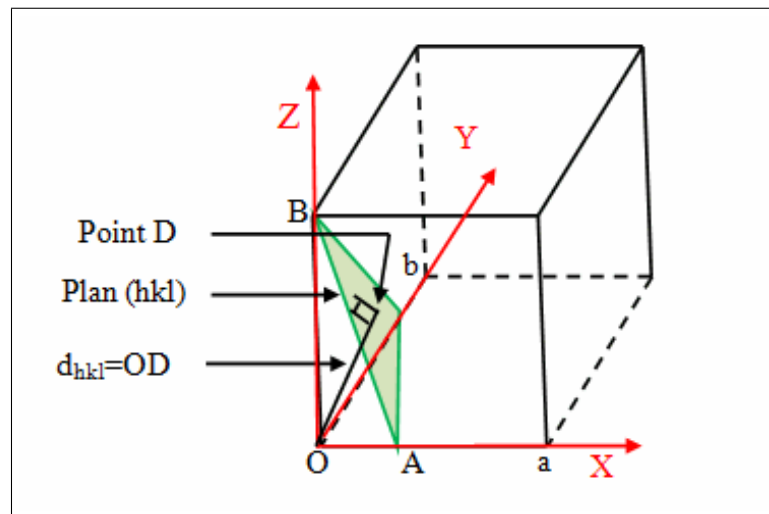


Figure 4.12: Lattice plane.

Examples: Lattice plain (See figure 4.13, Figure 4.14);

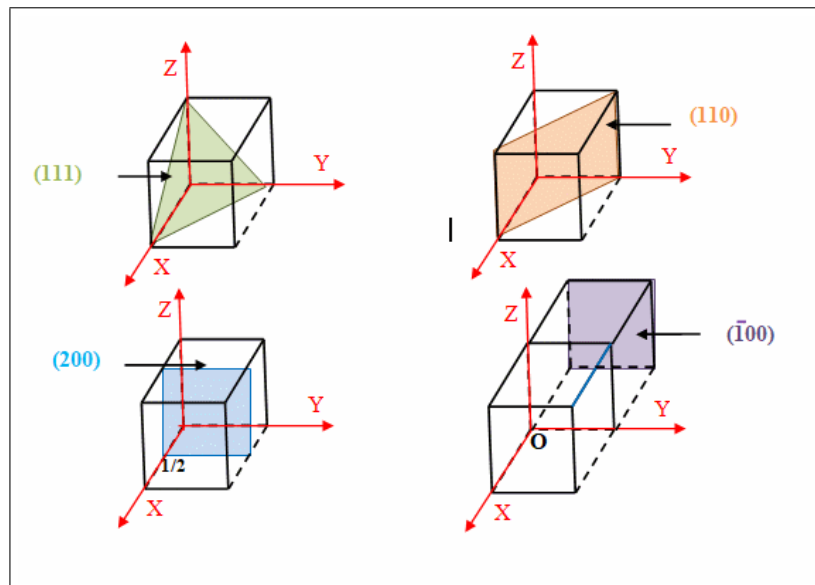


Figure 4.13: Example 1 (Lattice plane).

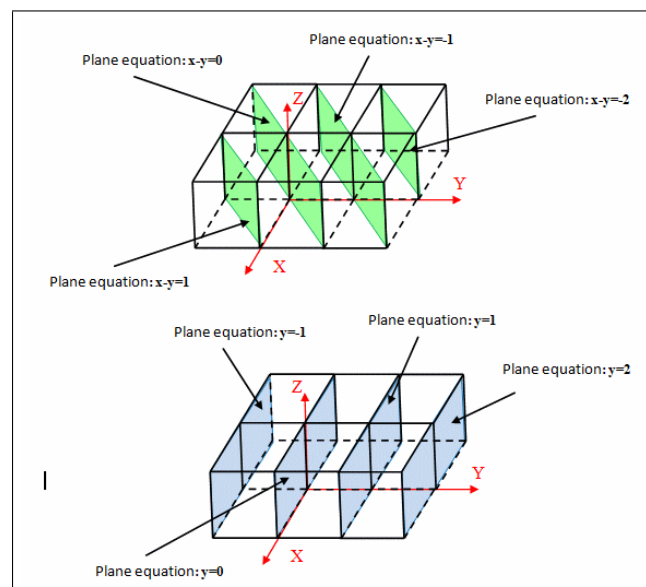


Figure 4.14: Example 2 (Plane equation).

4.11 Multiplicity

Multiplicity is the number of motifs per unit cell (See Figure 2.15);

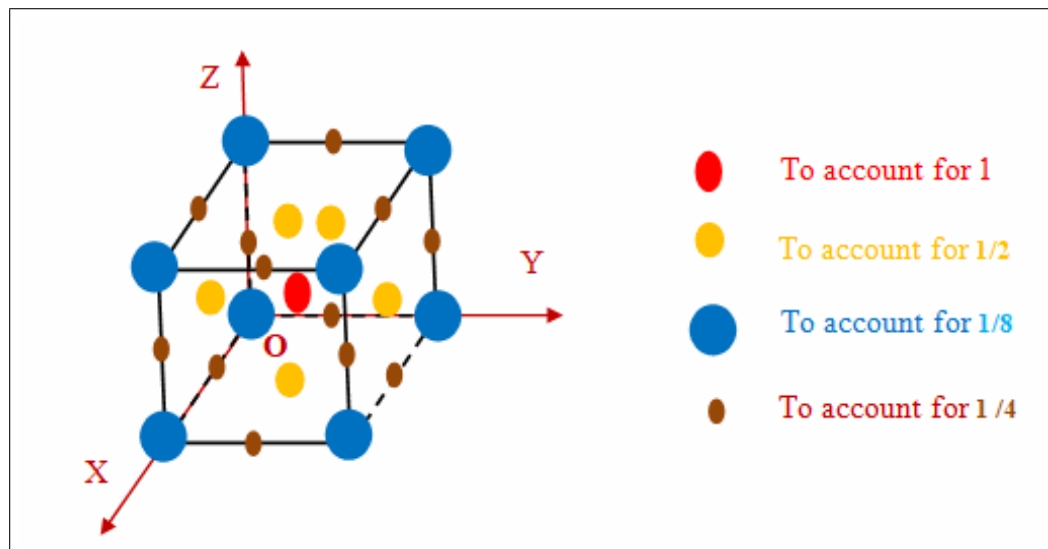


Figure 2.15: Multiplicity.

4.12 Bravais Lattices

* The Bravais lattices represent the 14 unique ways to arrange atoms in a three-dimensional crystalline lattice. They are categorized based on the symmetry of the unit cell, and they provide a fundamental framework for describing all possible crystal structures.

* The Bravais lattices are important because they allow scientist to classify crystal structures. This classification makes it easier to understand and predict the properties of crystalline materials.

* **Auguste Bravais** showed that the number of possible crystal systems was very limited. He listed 14 types of lattices which are variations of only 7 crystal systems.

* The Bravais lattices are: Triclinic, Monoclinic, Orthorhombic, Tetragonal, Rhombohedral, Hexagonal, and Cubic. each of those categories have different variations on them;

Cubic: (Figure 4.16).

$$a = b = c, \quad (4.9)$$

$$\alpha = \beta = \gamma = 90^\circ. \quad (4.10)$$

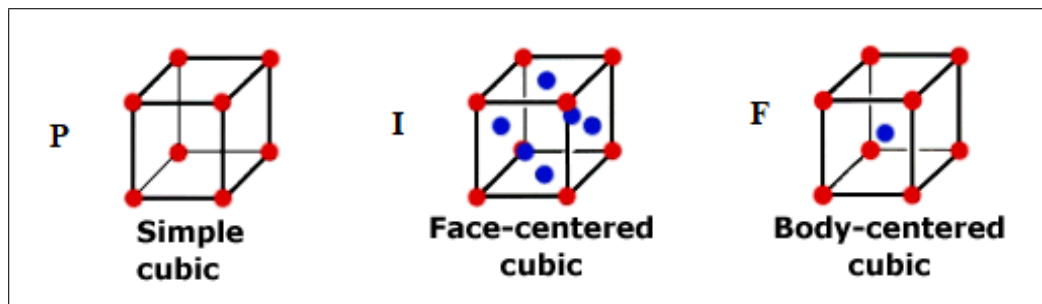


Figure 4.16: Cubic.

Tetragonal: (Figure 4.17)

$$a = b \neq c, \quad (4.11)$$

$$\alpha = \beta = \gamma = 90^\circ. \quad (4.12)$$

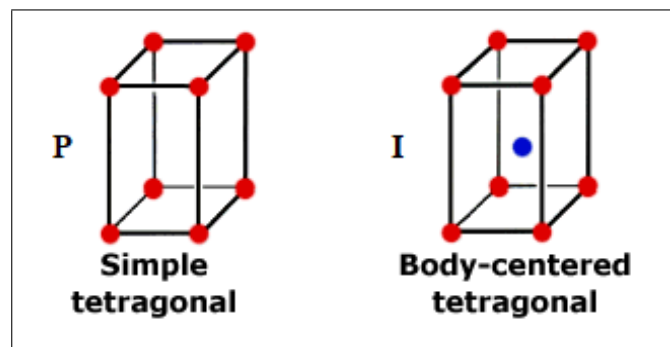


Figure 4.17: Tetragonal.

Orthorhombic: (Figure 4.18).

$$a \neq b \neq c, \quad (4.13)$$

$$\alpha = \beta = \gamma = 90^\circ. \quad (4.14)$$

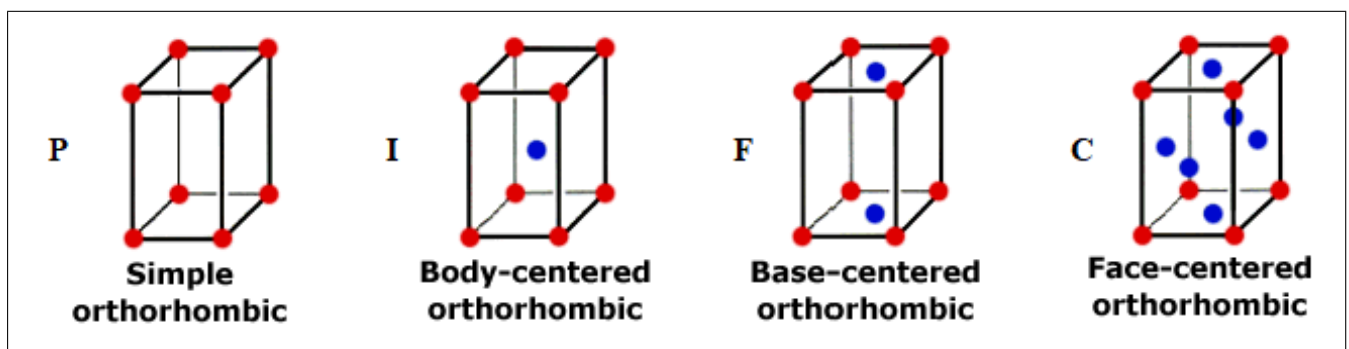


Figure 4.18: Orthorhombic.

Hexagonal: (Figure 4.19).

$$a = b \neq c, \quad (4.15)$$

$$\alpha = \beta = 90^\circ, \quad (4.16)$$

$$\gamma = 120^\circ. \quad (4.17)$$

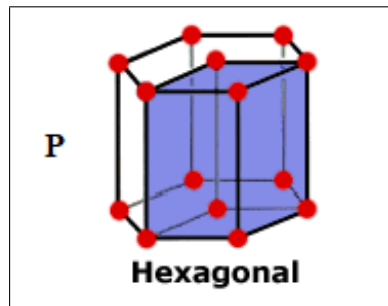


Figure 4.19: Hexagonal.

Monoclinic: (Figure 4.20).

$$a \neq b \neq c, \quad (4.18)$$

$$\alpha = \gamma = 90^\circ, \quad (4.19)$$

$$\beta \neq 120^\circ. \quad (4.20)$$

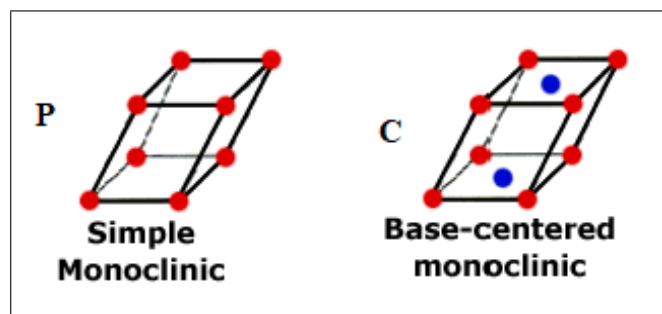


Figure 4.20: Monoclinic.

Triclinic: (Figure 4.21).

$$a \neq b \neq c, \quad (4.21)$$

$$\alpha \neq \beta \neq \gamma \neq 90^\circ, \quad (4.22)$$

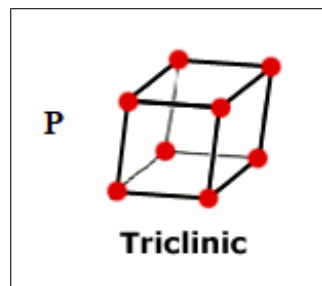


Figure 4.21: Triclinic.

Trigonal: (Figure 4.22).

$$a = b = c, \quad (4.23)$$

$$\alpha = \beta = \gamma \neq 90^\circ. \quad (4.24)$$

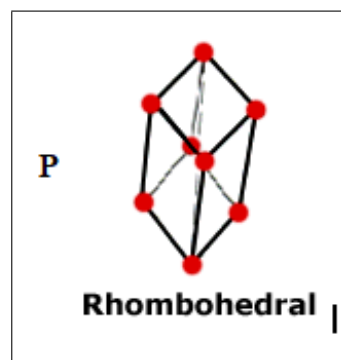


Figure 4.22: Trigonal.

Symbols:

P : **Primitive**: simple unit cell.

F : **Face-centered**: additional point in the center of each face.

I : **Body-centered**: additional point in the center of the cell.

C : **Base-Centered**: additional point in the center of each end.

R : **Rhombohedral**: Hexagonal class only.

4.13 Exercise

Exercise 1:

Considering an orthonormal coordinate system (O, x, y, z) .

Represent the following lattice planes.

$$(1, 1, 1), (1, 3, 1), (1, \bar{1}, 1), (1, 2, \bar{2}), (1, 1, 0), (\bar{2}, 0, 0). \quad (4.25)$$

Exercise 2:

Considering an orthonormal coordinate system (O, x, y, z) .

Represent the rows:

$$[110], [111], [1\bar{1}2]. \quad (4.26)$$

Exercise 3:

Let a family of planes (hkl) containing the following nodes:

$$\left(\frac{1}{2}, 0, 0\right); \left(0, \frac{2}{3}, 0\right); \left(0, 0, \frac{3}{4}\right).$$

Determine the Miller indices h, k, l of this family of planes.

4.14 Solutions

Exercise 1:

Representation of the lattice planes;

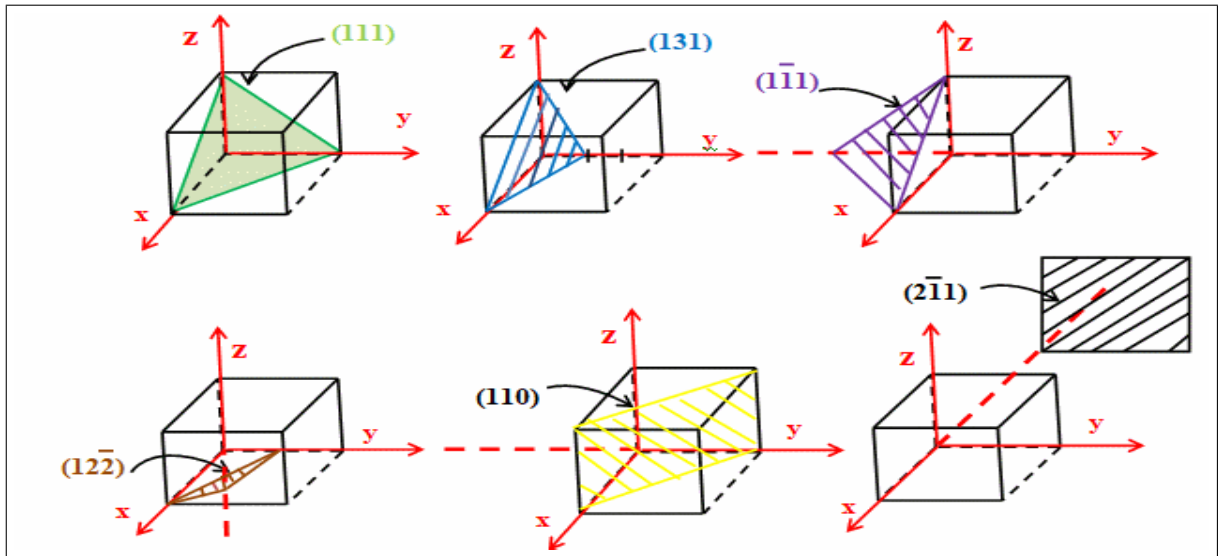


Figure 4.23: Lattice planes.

Exercise 2:

Representation of the rows;

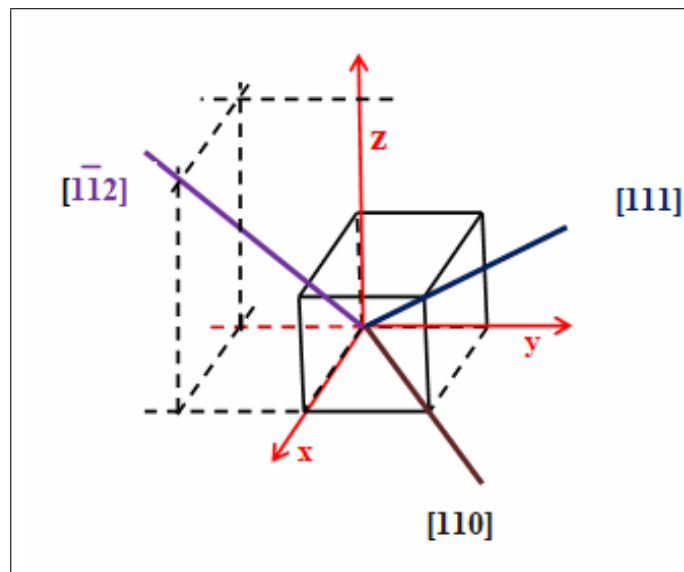


Figure 4.24: Rows representation.

Exercise 3:

Determination of the Miller indices h, k, l of the family of planes;

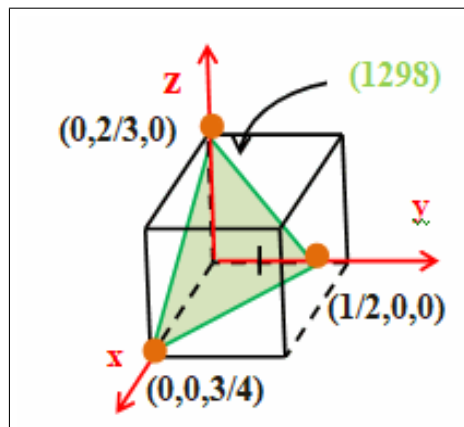


Figure 4.25: Miller indices.

Chapter 5

Concept of spectral analysis

5.1 Introduction

The objective of this chapter is to introduce fundamental concepts and principles of spectral analysis, encompassing the spectrum of light across its various ranges. Subsequently, we will explore the interaction between light and matter, its classification, and its correlation with wavelength. Ultimately, we aim to analyze basic light spectra and identify some of their applications and benefits.

5.2 Spectroscopy

Spectral analysis, or spectroscopy, is a scientific technique that studies the interaction between electromagnetic radiation and matter. It involves decomposing radiation (light, radio waves, X-rays, etc.) into its spectral components, i.e., according to their wavelength, frequency, or energy. This decomposition yields a spectrum, which is a kind of fingerprint of the analyzed matter.

Spectroscopy techniques are divided into atomic and molecular spectroscopy;

- * Atomic spectroscopy studies the energy and matter interactions between atoms.
- * Molecular spectroscopy studies the interaction of energy and matter between molecules

It is used in many fields: chemistry, physics, biology, astronomy, environment, medicine, etc.

5.3 Spectrophotometer

The study of the interaction between matter and light is usually done using what is called a Spectrophotometer, which typically consists of: light source, monochromator, cuvette to place a sample, photoelectric detector (Figure 5.1).

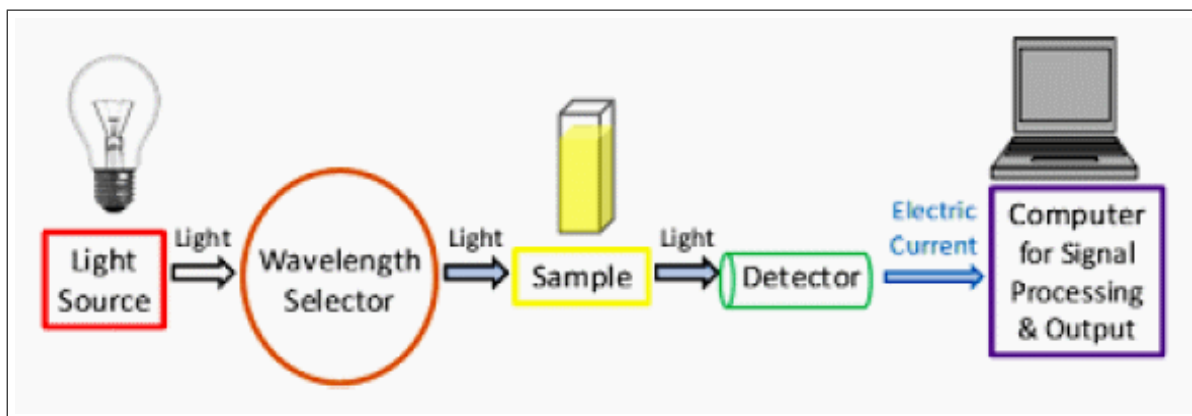


Figure 5.1: Spectrophotometer.

5.4 Spectrum

A spectrum is the representation of the characteristic wavelengths of electromagnetic radiation **emitted** or **absorbed** by a substance.

5.4.1 Types of spectrum

There are two types of spectrum:

* **Continuous (band spectrum):** A continuous spectrum is a light spectrum where the intensity of radiation varies continuously, without interruption, with a gradual transition from one wavelength to another.

Example: Sunlight spectrum (Figure 5.2).

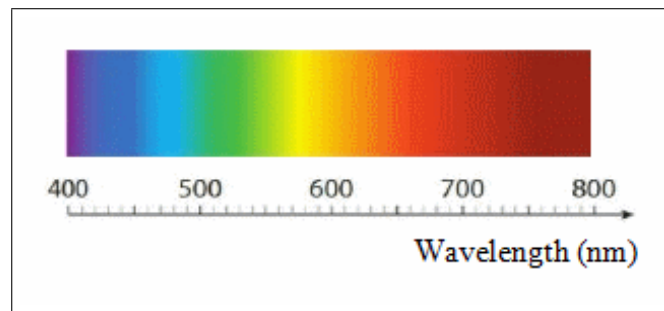


Figure 5.2: Sunlight spectrum.

* **Discontinuous (line spectrum):** A discontinuous spectrum, or line spectrum, is a light spectrum composed of distinct lines or bands, corresponding to specific wavelengths.

Example: Mercury emission spectrum (Figure 5.3).

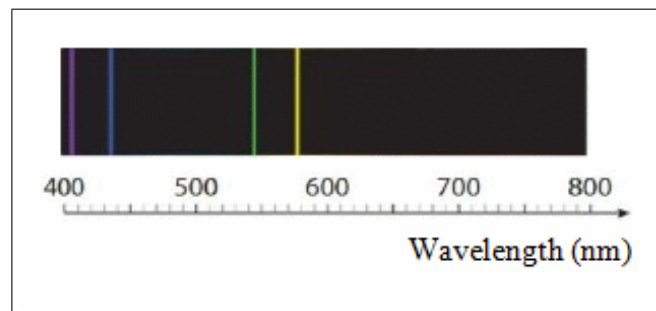


Figure 5.3: Mercury emission spectrum.

5.4.2 Electromagnetic spectrum

* It is the distribution of all electromagnetic waves, classified by wavelength or frequency.

* It contains: radio waves, microwaves, infrared, visible, ultraviolet, X-rays and gamma rays (Figure 5.4).

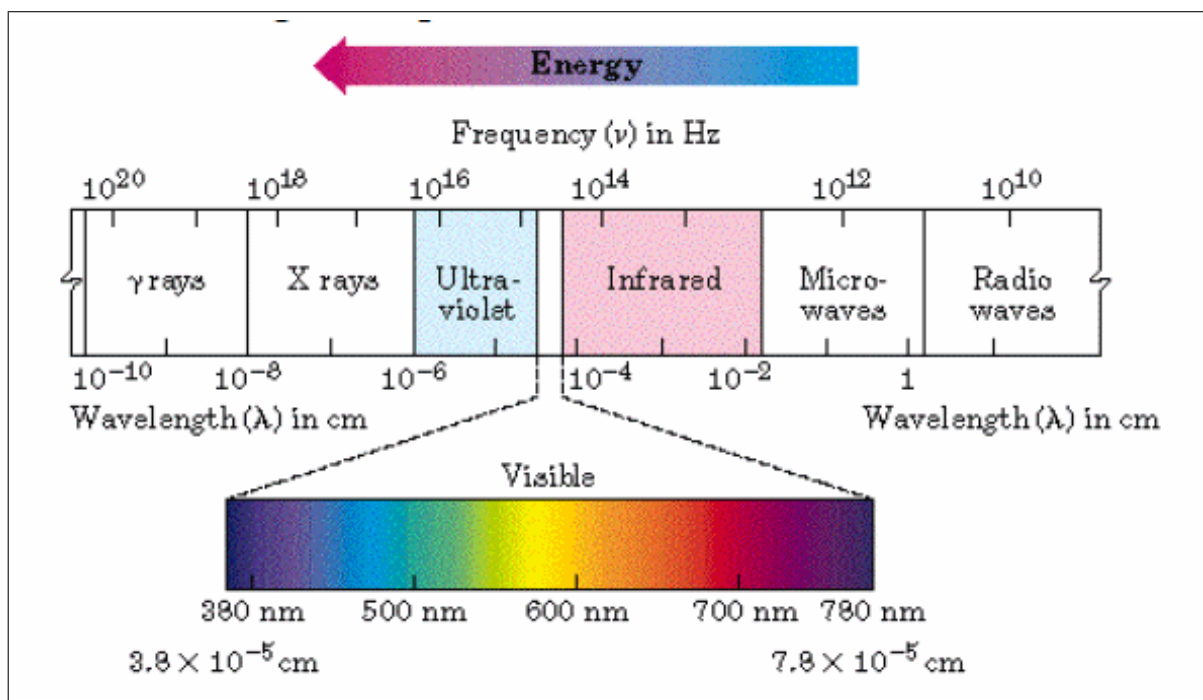


Figure 5.4: Electromagnetic waves

5.5 Wave-matter interaction

The interaction of electromagnetic radiation with matter can take different forms: absorption, emission, and scattering (See Figure 5.5).

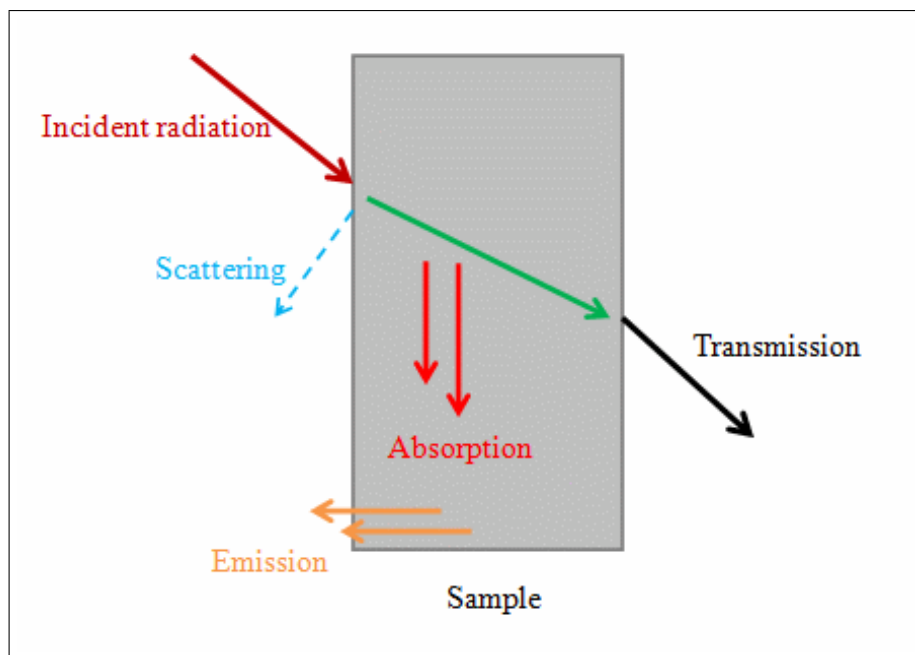


Figure 5.5: Basic types of wave-matter interaction .

5.5.1 Absorption:

Matter absorbs a photon, which increases its energy (Figure 5.6).

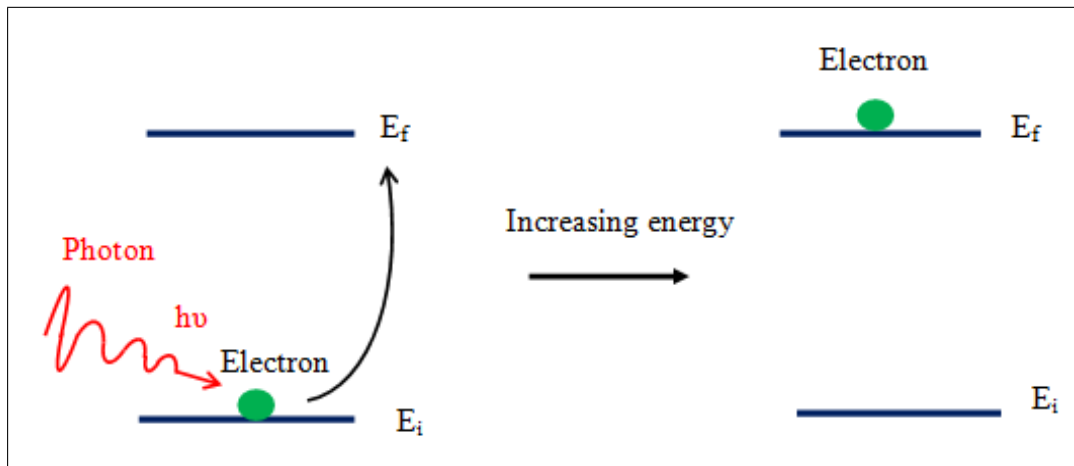


Figure 5.6: Absorption.

5.5.2 Emission:

Matter emits a photon by losing energy (Figure 5.7).

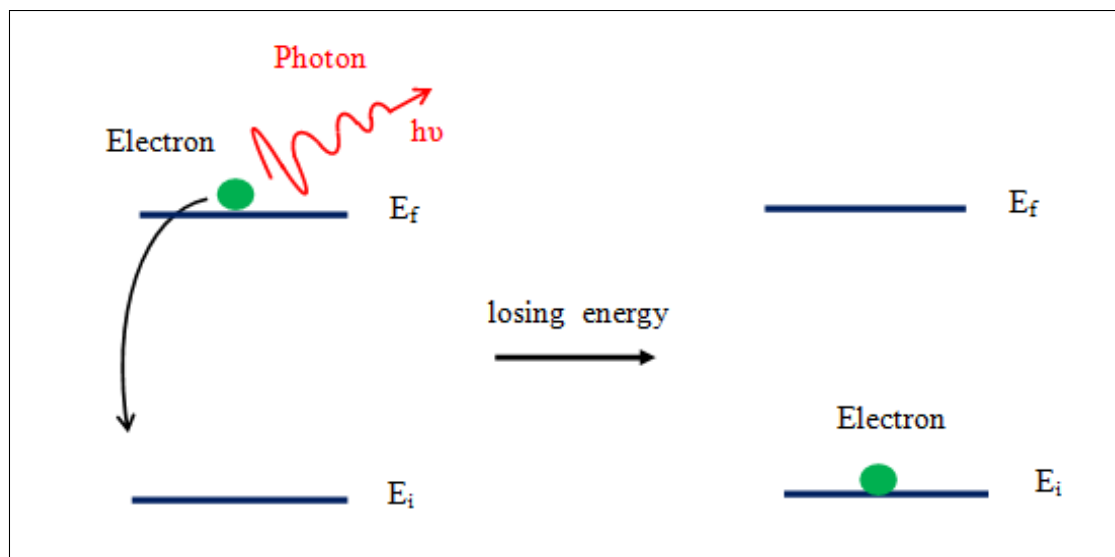


Figure 5.7: Emission.

5.5.3 Scattering:

Matter deflects the path of photons.

When light and matter interact, the system's energy can change. It can either increase (by gaining energy) or decrease (by losing energy). This energy change, called ΔE , happens when a tiny packet of light, a photon, is either absorbed (system gains energy) or emitted (system loses energy). The frequency (ν) of this photon is directly related to the energy change by the Planck-Einstein relationship:

$$\Delta E = E_f - E_i = h\nu, \quad (5.1)$$

where

$$\lambda = \frac{c}{\nu}, \quad (5.2)$$

with

$h = 6.626 \times 10^{-34} J.s$: Planck constant .

ν : Frequency of the radiation.

c : Velocity of light in a vacuum.

E_f : Final energy of the system.

E_i : Initial energy of the system.

Note:

* For **absorption**: $\Delta E > 0$, $E_f > E_i$.

* For **emission**: $\Delta E < 0$, $E_f < E_i$.

5.6 Law of light absorption: Beer-Lambert law

This fundamental law relates the absorption of light by a substance to its concentration.

Consider a monochromatic light passing through a solution (sample) of concentration $[c]$ in a cuvette of width l (Figure 5.8).

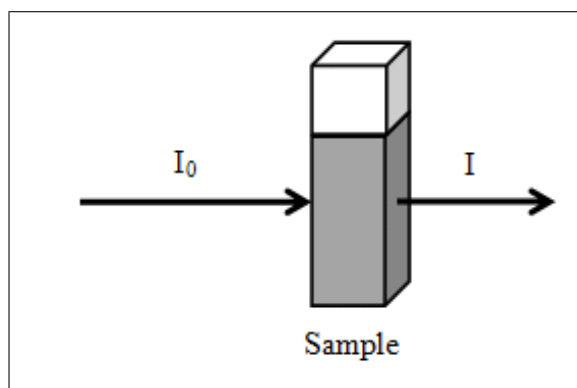


Figure 5.8: Beer-Lambert experiment.

Part of this radiation will be **absorbed** by the sample and part will be **transmitted** following the law;

$$I = I_0 e^{-klc}. \quad (5.3)$$

where

I_0 : Intensity of incident light

I : Transmitted intensity

l : Cuvette width

c : Concentration of absorbing species

k : Characteristic constant of the sample

The **absorbance** A is given by **Beer-Lambert's** law:

$$A = -\log T = \varepsilon.l.c. \quad (5.4)$$

where

T : Transmittance ($T = \frac{I_0}{I}$).

ε : Molar extinction coefficient (Characteristic of the substance at a given λ)

5.7 Spectroscopic methods:

The common spectroscopic methods based on electromagnetic radiation are summarized in the following table:

Type of spectroscopy	Usual wavelength range	Type of quantum transition
Gamma-rays emission	$0.005 - 1.4 \text{ \AA}$	Nuclear
X-rays absorption, emission fluorescence and diffraction	$0.1 - 100 \text{ \AA}$	Inner electron
Vacuum ultraviolet absorption	$10 - 180 \text{ nm}$	Bonding electrons
Ultraviolet-visible absorption	$180 - 780 \text{ nm}$	Bonding electrons
Infrared absorption and Raman scattering	$0.78 - 300 \text{ }\mu\text{m}$	Rotation/ Vibration of molecules
Microwaves absorption	$0.73 - 376 \text{ mm}$	Rotation of molecules
Electron spin resonance	3cm	Spin of electrons in a magnetic field
Nuclear magnetic resonance	$0.6 - 10\text{m}$	Spin of nuclei in a magnetic field

5.8 Exercise

Exercise 1:

Here is the energy level diagram of the hydrogen atom (Figure 5.9):

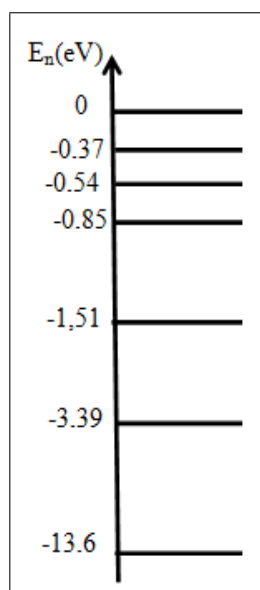


Figure 5.9: Energy diagram of the hydrogen atom.

1/ What is the value of the ground state energy of the hydrogen atom?

- 1/ What is the value of the energy in eV required to ionize this atom?
- 3/ Calculate the change in energy in eV and in Joules when the hydrogen atom transitions from the energy level $E_1 = -0.37eV$ to the energy level $E_2 = -3.39eV$.
- 4/ On the diagram above, represent this transition with an arrow.
- 5/ Is this photon released or absorbed by the hydrogen atom?
- 6/ Calculate the wavelength λ of the photon corresponding to this energy change.

Exercise 2:

A spectroscope provides the spectrum of an atomic vapor lamp that contains a gas enclosed in a glass bulb (Figure 5.10).

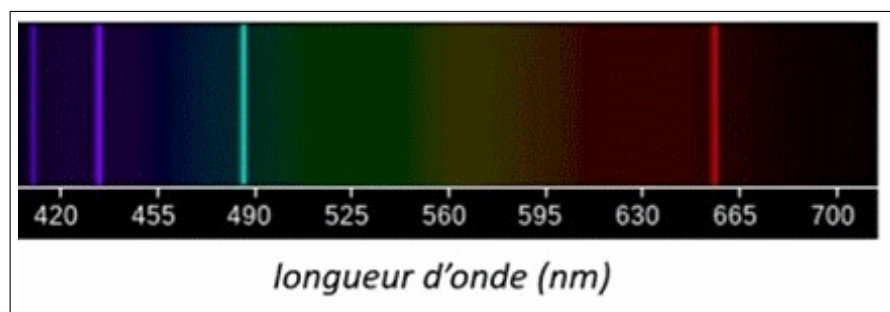


Figure 5.10: Spectrum of an atomic vapor lamp.

- 1/ What type of spectrum is it?
 - 2/ Identify the entity responsible for the light emission.
- Given; wavelengths (in nm) of some characteristic radiations.

Hydrogen	410, 434, 486, 656
Lithium	412, 496, 610, 671
Mercury	405, 436, 546, 579

Exercise 3:

A spectrometer was used to plot the spectrum of an orange solution of potassium dichromate with a concentration $C_0 = 0.6mol/L$ as shown in the figure below (Figure 5.11). A calibration table was then created by measuring the absorbance A for different concentrations of dichromate ions. The spectrophotometer was used with a wavelength $\lambda = 450nm$.

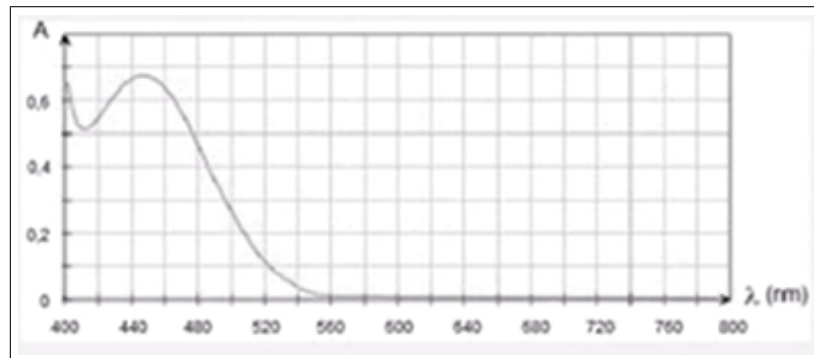


Figure 5.11: Spectrum of an orange solution of potassium dichromate with a concentration $C_0 = 0.6 \text{ mol/L}$.

1/ Why is the wavelength $\lambda = 450 \text{ nm}$ used?

2/ Plot the curve $A = f(C)$?

3/ Is Beer-Lambert Law verified?

4/ We have an orange potassium dichromate solution of unknown concentration C_1 . It is diluted 10 times. The absorbance of the diluted solution is measured and found to be $A_2 = 1.60$. Calculate the concentration C_2 of the diluted solution and then the concentration C_1 of the initial solution.

5.9 Solutions

Exercise 1:

1/ The ground state energy (E_0) of the hydrogen atom:

$$E_0 = -13.6 \text{ eV}. \quad (5.5)$$

2/ The value of the energy in eV required to ionize this atom;

$$E = 13.6 \text{ eV}. \quad (5.6)$$

3/ The change in energy in eV and in Joules when the hydrogen atom transitions from the energy level $E_1 = -0.37 \text{ eV}$ to the energy level $E_2 = -3.39 \text{ eV}$;

$$\Delta E = -3.39 \text{ eV} - (-0.37 \text{ eV}) = -3.02 \text{ eV} = -3.02 \times 1.6 \times 10^{-19} \text{ J} = -4.8 \times 10^{-19} \text{ J}. \quad (5.7)$$

Where $1\text{eV} = 1,6 \times 10^{-19}\text{J}$.

4/ Representation of this transition with an arrow (Figure 5.12):

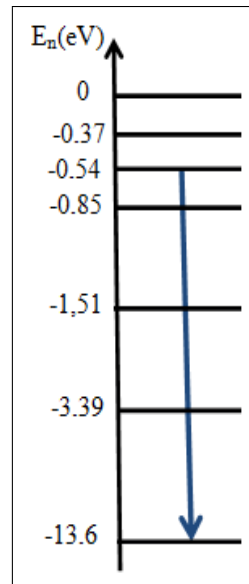


Figure 5.12: Hydrogen atom transitions.

5/ The photon is released by the hydrogen atom.

6/ The wavelength λ of the photon corresponding to this energy change:

$$\lambda = \frac{hc}{|\Delta E|} = \frac{6,626 \times 10^{-34} \text{J.s} \times 3 \times 10^8 \text{m/s}}{4,8 \times 10^{-19} \text{J}}. \quad (5.8)$$

$$= 4.1412510^{-7} \text{m} = 414,12 \text{nm}. \quad (5.9)$$

Exercise 2:

1/ This is a line spectrum, characteristic of the light emitted by an atomic gas.

2/ By exploiting the emission line spectrum, we identify:

- A radiation below 420nm (hydrogen, lithium, and mercury).
- A radiation between 420nm and 450nm (hydrogen and mercury).
- A radiation close to (below) 480nm (hydrogen).

The gas being sought is hydrogen because it is the only gas that possesses these characteristic radiations in its line spectrum.

Exercise 3:

1/ We use the wavelength $\lambda = 450\text{nm}$ because the absorbance A is greatest at this wavelength, as can be seen here in the graph.

2/ The curve $A = f(c)$ (Figure 5.13).

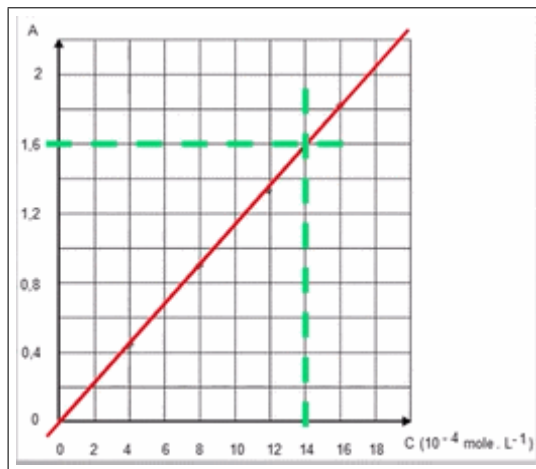


Figure 5.13: The curve $A = f(c)$.

3/ Verification of the Beer-Lambert law.

The curve $A = f(c)$ is the equation of a linear line of the form

$$A = K \times C \quad (5.10)$$

Such that K is the slope of the straight line then the Beer-Lambert law verified.

4/ Calculation of C_1 and C_2 .

$$C_1 = 10C_2 = 10 \times 1.4 \times 10^{-2} = 0.14 \text{ mol/l.} \quad (5.11)$$

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